

## Rheology of Concentrated Suspensions

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### ABSTRACT

There have been many approaches taken to clarify and to explain the flow behavior of suspensions. These approaches may be classified into three categories, the first of which is a group of works to establish non-Newtonian viscosity equation(s) based on the famous Einstein equation (1).

$$\eta_s = \eta_m(1 + 2.5-Cv) \dots\dots\dots(1)$$

This approach have not been successful due to the problems associated with the original equation. Since Eq. (1) does not contain parameters describing the time-dependency, the shear-dependency, and specific properties of particles, it is not suitable as a fundamental equation describing non-Newtonian viscosity.

The second category contains works to establish non-Newtonian viscosity equation(s) based on the destruction of structures of coagulated particles or on the decreasing number of junctions between particles. Although many equations have been proposed, none of them have been accepted as a general viscosity equation for a variety of suspensions. One of the serious problems associated with this approach may be a lack of an appropriate considerations on the increasing number of junctions due to the naturally occurring coagulation.

The third category is a classification of non-Newtonian fluids by the flow types based on the shapes of experimental flow curves, and to define each type by flow equations expressing flow curves. Two serious problems are associated with this approach and one of which is that there are too many types of flow curves to be classified, and many of the flow equations proposed in the past do not satisfy the rule of physical dimension. Another problem in this category is that many of the flow equations proposed do not express the time-dependency. Without defining the shearing conditions as functions of time, there are possibilities of classifying a fluid differently since a fluid performs differently under different conditions.

To solve problems mentioned as above, the authors have tried to establish a new rheological theory by combining a classical concept that the viscosity of fluids was originated from the friction between the components, the Verwey-Overbeek theory describing the increase of junction number by natural coagulation, and the concept of structural viscosity theory in which the the viscosity decrease was attributed to the break-down of coagulated structure of particles, and obtained a general viscosity equation (2) which was capable of describing the time-dependency and the shear-dependency.

$$\eta_3 = B_3 \cdot n_3^{2/3} \left\{ \frac{U_0(\gamma H t^2 + 1) + H t}{(H t + 1)(\gamma t + 1)} \right\}^{2/3} \dots\dots\dots(2)$$

- $\gamma$  : shear rate
- $\eta_3$  : viscosity originated from friction between particles
- $B_3$  : coefficient of friction between two coagulated particles
- $n_3$  : number of primary particles in unit volume of suspension
- $U_0$  : parameter describing the initial state of dispersion at the start of rheological experiment
- $t$  : experimental time (0 at the start of experiment or observation)
- $H$  : coagulation rate constant of particles

By introducing conditions into Eq. (2), a variety of expressions are obtainable. For example, if the shearing condition in which the shear rate increases proportionally to time as expressed by Eq. (3) is introduced, Eq. (2) is transformed to Eq. (4) which expresses the viscosity change under the condition defined by Eq. (3).

$$\gamma = G \cdot t \dots\dots\dots(3)$$

- $G$  : non-dimensional constant describing the shearing condition

$$\eta_3 = B_3 \cdot n_3^{2/3} \left\{ \frac{U_0(G H t^3 + 1) + H t}{(H t + 1)(G t^2 + 1)} \right\}^{2/3} \dots\dots\dots(4)$$

If the suspension is left unagitated, the shear rate is zero, and Eq (2) is transformed to Eq. (5) to express the viscosity increase of unagitated suspensions.

$$\eta_3 = B_3 \cdot n_3^{2/3} \left( \frac{U_0 + H t}{H t + 1} \right)^{2/3} \dots\dots\dots(5)$$

In agitated suspensions, natural coagulation and mechanical dissociation of particles are taking place simultaneously and there is a time of equilibrium at which the rates of coagulation and of dissociation become equal. The junction number and the viscosity of suspension become constant after the time of equilibrium. The time of equilibrium ( $t_e = 1/\sqrt{\gamma \cdot H}$ ) is calculable from the condition to make the derivative of Eq. (2) equal to zero, and by substituting  $t$  in Eq. (2) by  $t_e$ , we have Eq. (6) for expressing constant viscosity of suspensions.

$$\eta_3 = B_3 \cdot n_3^{2/3} \left\{ \frac{2 \cdot U_0 \cdot \sqrt{\gamma \cdot H} + H}{(\sqrt{H} + \sqrt{\gamma})^2} \right\}^{2/3} \dots\dots\dots(6)$$

There are many ways of application of Eq. (2), the general viscosity equation, and some examples including calculations and analysis of flow curves are presented here.