Strength Analysis of Yttria-Stabilized Tetragonal Zirconia Polycrystals

Kenichi Noguchi and Manabu Oishi
Toray Research Center, Imc., Otsu, Shiga, 520, Japan
Takaki Masaki
Toray Industries, Inc., Otsu, Shiga, 520, Japan

ABSTRACT

Tensile strength of Y_2O_3-stabilized ZrO_2 polycrystals(Y-TZP) was measured by a newly developed tensile testing method with a rectangular bar. The tensile strength of Y-TZP was lower than that of three-point bend strength, and the shape of the tensile strength distribution was quite different from that of three-point bend strength distribution.

It was quite difficult to predict the distribution curve of the tensile strength using the data of the three-point bend strength by one-modal Weibull distruibution. The distribution of the tensile strength was analyzed by two-or-three-modal Weibull distribution coupled with an analysis of fracture origins.

The distribution curve of the three-point bend strength which was estimated by multimodal Weibull distribution agreed favorably with that of the measured three-point bend strength values.

A two-modal Weibull distribution fuction was formulated approximately from the distribution of the tensile and three-point bend strengths, and the estimated two-modal Weibull distribution fuction for the four-poin bend strength agreed well with the measured four-poit bend strength.

I. Introduction

Design of ceramic components in structural applications is necessary to assure their mechanical reliability and safety. The fracture of brittle material is controlled by defects which populate the specimen. The strength of brittle material show large valiability and dependence on the size of the specimen. The probabilistic characteristics of strength are generally analyzed by a Weibull distribution funtion[1-3].

The strength of ceramic materials has usually been represented by the bend strength value, since specimen preparation and testinf are easily conducted. However, In the bend test, stress gradients exist in the specimen, and effective volume[2-3] is quite small compared withthat of the tensile test. Therefore, using bend strength data to design mechanical parts has many limitations.

Although many tensile test method have been already proposed[4-7], they are considered to be difficult to perform because of complicated specimen shape, special test fixture used, and stress concentration and eccentricity encountered during the testing. Therefore, far fewer data are reported for tensile strength than bend strength in ceramic materials.

This paper reports the tensile strength data of Y-TZP materials tested by a newly developed method with a rectangular bar[8]. The statistical variations of the tensile and three-point bendstrengths are quite different from each other and may be quantiatatatively analyzed by a multimodal Weibull distribution. An approximate formation for such a two-modal Weibull distribution function obtained from the measured tensile and three-point bend strengths is described.

II. Experimental Procedure

TZPs containing 2.0, 2.5, and 3.0 mol% Y_2O_3 and 0.1 and 0.5 wt% Al_2O_3 were formulated. The materials studied are shown in Table I. The powder of sample A was prepared by the thermal decomposition method[9]. Sample A was prepared by pressureless sintering and hot isostatic pressing methods as previously reported[9].

The sintered material was cut with a diamond saw and ground with a 400-grit diamond wheel to the dimensions 10 mm by 1 mm by 170 mm, and 4 mm by 3 mm by 36 mm, for the tensile and bend tests, respectively.

The three-point bend test was conducted using a support spun of 30 mm and a crosshead rate of 0.5 mm/min. The four-point bend test was conducted using a support spun of 30 mm and a load spun of 10 mm. The crosshead rate of the four-point bend test was 0.5 mm/min.

The tensile test was performed using the specimen bonded by glass-fiber-reinforced plastic(GFRP) tabs with an epoxy adhesive on the rectangular TZP plate, as shown in Fig. 1[6]. To ensure alignment of the tensile specimen as it was fractured, two strain gages (A and A) were attached on one side, and a third strain gage (B) was attached on the other side. The tensile test was performed using a universal test machine at a crosshead rate of 0.5 mm/min. All strength tests were done at 23°C and 50% relative humidity. Fracture surface was examined with scanning electron microscopy and electron probe microanalysis (SEM-EPMA).

III. Results and Discussion

(1) Tensile Test

Figure 2 shows the stress-strain curves of the tensile test specimen attached to three-strain gages (A_1, A_2, and B). Similar curves are obtained. Table II shows the tensile breaking strain of sample A, where the breaking strains ε_{A1}, ε_{A2}, and ε_B are obtained by three straingages, A_1, A_2, and B, respectively.

The stress eccentricities in the direction of depth and width of tested specimens are small, since the mean values of $|\varepsilon_{A1} - \varepsilon_{A2}| / |\varepsilon_{A1} + \varepsilon_{A2}|$ and $|\varepsilon_A - \varepsilon_B| / |\varepsilon_A + \varepsilon_B|$ are 0.03 and 0.01, respectively. Therefore, the effect of bending mode on the tensile strength will be small[6].

(2) Tensile and Bend Strengths

Test results of tensile and three-point bend strengths of sample A are shown in Table III. The mean value of the tensile strength is much lower than that of the three-point bend strength, and the coefficient of the tensile strength is larger than that of the three-point bend strength.

The strength of brittle material is often analyzed by the Weibull distribution function[1-3]. The fracture probability can be given as

$$F(\sigma) = 1 - \exp\left[- V \left(\frac{\sigma}{\sigma_0} \right)^m \right] \qquad (1)$$

Where $F(\sigma)$ is the fracture probability to a stress σ, σ_0 the scale parameter, V the effective volume[3], and m the shape parameter known as the Weibull modulus.

Figure 3 shows the Weibull plots of the tensile and three-point bend strengths for sample A. The following estimator is used to calculate the fracture probability, F[2,3].

$$F = \frac{i - 0.5}{n} \qquad (2)$$

where i is the rank of the the strength value and n the total number of the specimens.

A distribution of the three-point bend strength can be expressed approximately by the dotted line, I, of the one-modal Weibull fdistribution function $F_B(\sigma)$ and then given by replacing the exponential term in Eq.(1) by $[-(\sigma/1650)^{12}]$.

The effective volume of the three-point bend test becomes 1.1 mm^3 from Eq. (A-4) shown in the Appendix, since the Weibull modulus m, is 12 and the volume, V_0 of the specimen between two support is 360 mm. The effective volume of the tensile test is also given as 500 mm^3.

The distribution funtion of the tensile strength $F_T(\sigma)$ estimated from $F_B(\sigma)$ can be expressed as $[-(\sigma/990)^{12}]$.

However, solid line, II, so calculated does not agree with the curve of the measured tensile strength values as shown in Fig.3. Although the distribution of the three-point bend strength appears to be a one-modal Weibull distribution function as shown in curve I, one could not simply predict the tensile strength of the material having a larger volume. Distribution curve III of the measured tensile strength cannot be simply expressed by one-modal Weibull distribution.

In the following section, the distribution curves of the tensile strength of sample A can be analyzed by multimodal Weibull distribution. The relationships between the tensile and bend strengths are examined.

(3) Strength Analysis by Multimodal Weibull Distribution

Fracture surfaces of the tensile test specimens are examined in order to identify the fracture origin by SEM-EPMA. Two types of fracture origins are classified in sample A. Figure 4 shows the two kinds of fracture origins, classified as inclusion and unknown. The fracture caused by the unknown type initiates not only at the surface but also inside of the specimen. Therefore, the fracture causes will be treated as a volumetric defect.

The individual fracture probabilities are estimated by the Johnson method[2,10], which determinesthe ranking number for the strength by calculating a new increment as soon as one or more censored strengths are encountered in the data.
This new increment, Δ, is

$$\Delta = \frac{(n+1) - i}{1 + J} \qquad (3)$$

where i is the previous ranking and J the number of specimens beyond the present censored set. The new ranking is then given by simply adding the calculated new increment to the previous ranking.

Figure 5 shows the Weibull plots for the individual strength distribution. Each strength distribution, F_{T1}, and F_{T2} is approximately by straight lines I and II where Eq. (1) may be rewritten such that the exponential term is given by $-(\sigma/930)^{2.7}$ and $-(\sigma/950)^{12.3}$, respectively. In a situation where the populations of the two defects exist concurrently, the overall fracture probability, F, is the product of the two individual fracture probabilities:

$$F(\sigma) = 1 - [1 - F_1(\sigma)][1 - F_2(\sigma)] \qquad (4)$$

where $F_1(\sigma)$ and $F_2(\sigma)$ are the fracture probabilities associated with the defectes of types 1 and 2, respectively. Equation (1) and (4) yield

$$F(\sigma) = 1 - \exp[-V_1(\frac{\sigma}{\sigma_{01}})^{m1} - V_2(\frac{\sigma}{\sigma_{02}})^{m2}] \qquad (5)$$

Substituting for F_{T1} and F_{T2} gives

$$F_T(\sigma) = 1 - \exp[-(\frac{\sigma}{930})^{2.7} - (\frac{\sigma}{950})^{12.3}] \qquad (6)$$

where $F_T(\sigma)$ is the two-modal Weibull distribution function the tensile strength of sample A.

Then, the strength distribution function given by Eq.(7) for sample A at the different effectve volumes can be obtained from Eqs.(5) and (6) when V_1 and V_2 equal 500 mm^3.

$$F_T(\sigma) = 1 - \exp[-V_1(\frac{\sigma}{9080})^{2.7} - V_2(\frac{\sigma}{1580})^{12.3}] \qquad (7)$$

The individual effective volumes for the three-point bend strength, V_1 and V_2, are calculated from Eq.(A-4) in the Appendix. Substituting the calculated V_1 and V_2 in Eq.(7) gives the distribution function of the three-point bend strength:

$$F_B(\sigma) = 1 - \exp[-(\frac{\sigma}{3560})^{2.7} - (\frac{\sigma}{1570})^{12.3}] \qquad (8)$$

Figure 6 shows the distribution curves calculated from Eqs. (6) and (8). The estimated distribution curves II for the three-point bend strength agrees well with the measured values. The different behaviour of the tensile and three-point bend strengths can be explained quantitatively by a two-modal Weibull distribution.

Figure 7 shows the strength distribution curves calculated from Eq. (7) for the various sizes of sample A under uniform tension. The effect of the inclusion having a small Weibull modulus increases as the volume of the specimens increases. On the other hand, the effect of the unknown origins having the larger Weibull modulus increases as the size of the specimen decreases. In both cases, the strength distribution function appear to be one-modal Weibull distributions, although the two Weibull modului are different from each other as shown in Eq. (7). Thus, a multimodal Weibull distribution can be easily misinterpreted as one-modal perfect population.

Therefore, it may be difficult to detect the dangerous defects with low frequency by the three-point bend test since the effective volume of a three-point bend test is very small. Both tensile and bend tests having different effective volumes can be utilized appropriately to realize the presize strength distribution, such as two-modal Weibull distribution.

Strength analysis by multimodal Weibull distribution gives quantitative suggestion of the improvements of materials. The distribution function, Eq. (6), for the tensile strength becomes F_{T2} when the inclusions are eliminated. Substituting the Weibull modulus m and scale parameter σ_0 of F_{T2} into Eqs. (A-2) and (A-3) in the Appendix gives the inproved mean value, $\bar{\sigma}$, of 910 MPa and a coefficient of variation, CV, of 10%, respectively.

IV. Conclusions

The strengths of Y-TZPs are quantitatively analyzed by a multimodal Weibuu distribution function. The following conclusions can be drawn from this study:

(1) Concurrent defects such as pores, machining flaws, agglomerates, cubic phase, silica and alumina inclusions, and alumina inclusions control the strength.

(2) It is dangerous to use the data of bend strength to predict the size effect.

Appendix

One-Modal Weibull Distribution Analysis

The strength of brittle material can be analyzed by the Weibull distribution function. The fracture probability can be given as

$$F(\sigma) = 1 - \exp\left[- V\left(\frac{\sigma}{\sigma_0} \right)^m \right] \qquad \text{(A-1)}$$

where $F(\sigma)$ is the fracture probability to a stress σ, σ_0 the scale parameter, V the effective volume, and m the shape parameter known as the Weibull modulus. The mean $\bar{\sigma}$ and the coefficient of variation of the strength, CV, are given by

$$\sigma = \sigma_0 V^{-1/m} \Gamma(\frac{1}{m}) \qquad \text{(A-2)}$$

$$CV = [\frac{\Gamma(1 + 2/m)}{\Gamma^2(1 + 1/m)} - 1]^{1/2} \times 100 \ (\%) \qquad \text{(A-3)}$$

where for a specimen under uniform tension, the effective volume V is the tested specimen volume.

In the case of the bend test, a stress gradient exists in the specimen. The effective volum e, V, for the three-point bend strength is given by

$$V = \frac{V_o}{2(m + 1)^2} \qquad \text{(A-4)}$$

where V_0 is the volume of the specimen between two supports. The effective volume of the four-point bend strength tested with a load spun of one-third of a support span is given by

$$V = \frac{m + 3}{6(m + 1)} V_0 \qquad \text{(A-5)}$$

where V_0 is the volume of a specimen between two supports.

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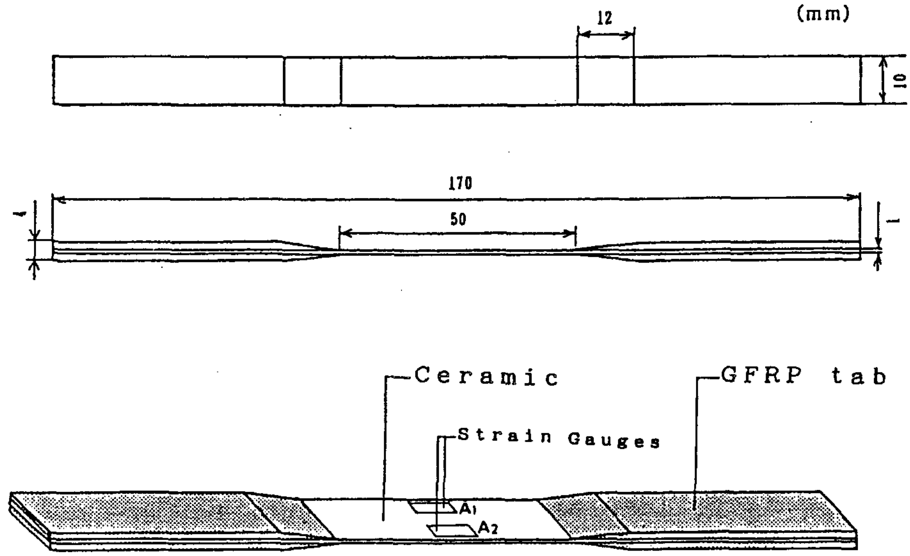


Fig.1 Tensile test specimen.

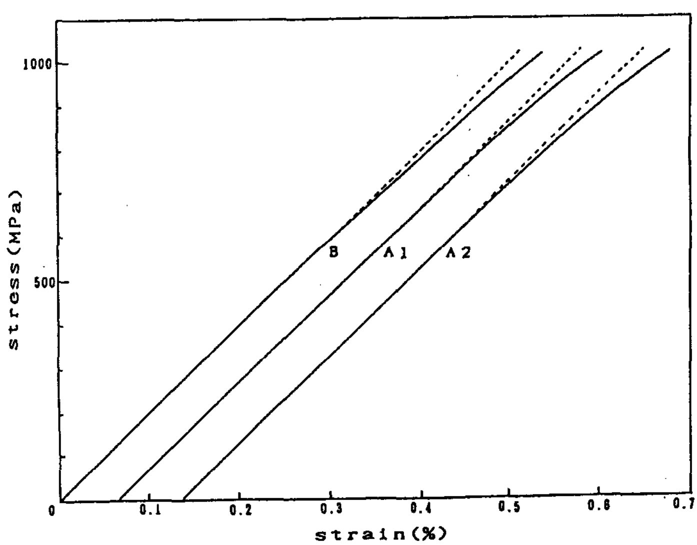


Fig.2 Stress-strain curves of sample A obtained by attaching three strain gages (A1, A2, and B).

Table I. Materials Used in Work

	Composition			Sintering temp. ($^{\circ}C$)		Density
Sample	Y_2O_3 (mol%)	Al_2O_3 (wt%)	Powder preparation method	PS*	HIP+	(g/cm)
A	2.0	0.5	Thermal decomposition	1400	1400	6.09

* PS is pressureless sintering. + HIP is hot isostatic pressing.

Table II. Breaking Strains of Sample A

Specimen No.	ε_{A1} (%)	ε_{A2} (%)	$\varepsilon = \frac{\varepsilon_{A1} + \varepsilon_{A2}}{2}$	ε_B (%)	$\frac{\lvert \varepsilon_{A1} - \varepsilon_{A2} \rvert}{\varepsilon_{A1} + \varepsilon_{A2}}$	$\frac{\lvert \varepsilon_A - \varepsilon_B \rvert}{\varepsilon_A + \varepsilon_B}$
1	0.459		(0.459)	0.460		(0.001)
2	0.417	0.393	0.405	0.393	0.03	0.02
3	0.512	0.464	0.488	0.488	0.05	0.000
4	0.306	0.280	0.293	0.296	0.04	0.005
5	0.524	0.512	0.518	0.517	0.01	0.001
6	0.464	0.441	0.453	0.445	0.03	0.002
7	0.126	0.114	0.120	0.116	0.05	0.02
8	0.443	0.422	0.433	0.427	0.02	0.007
9	0.391	0.391	0.391	0.386	0.000	0.006
10	0.282	0.284	0.283	0.289	0.003	0.01
Mean	0.39	0.37	0.38	0.38	0.03	0.01

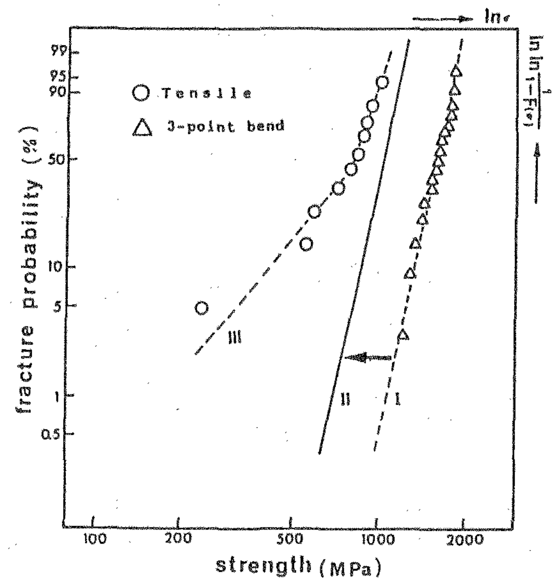
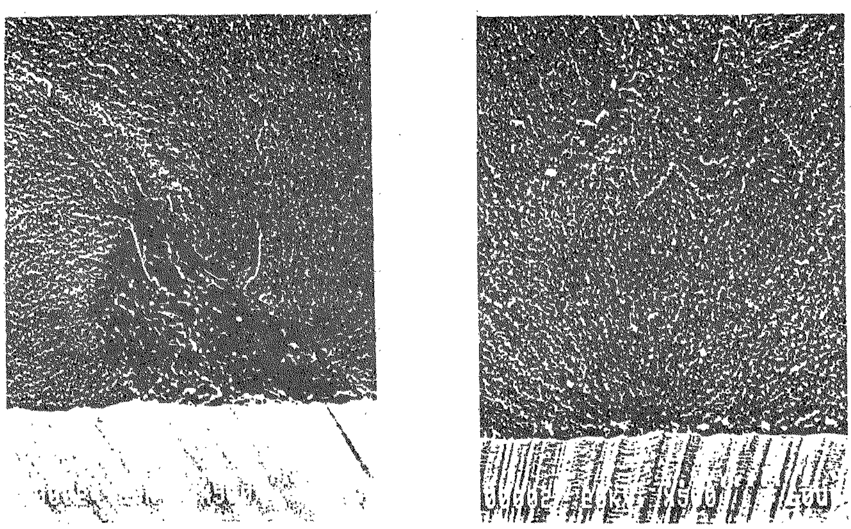


Fig. 3 Weibull plots for sample A. Dotted line I reveals the distribution of three-point bend strength given by $F_B(\sigma)$. Solid line II represents the tensile distribution calculated from $F_T(\sigma)$.



inclusion origin　　　　unknown origin

Fig. 4 Typical fracture origins of sample A.

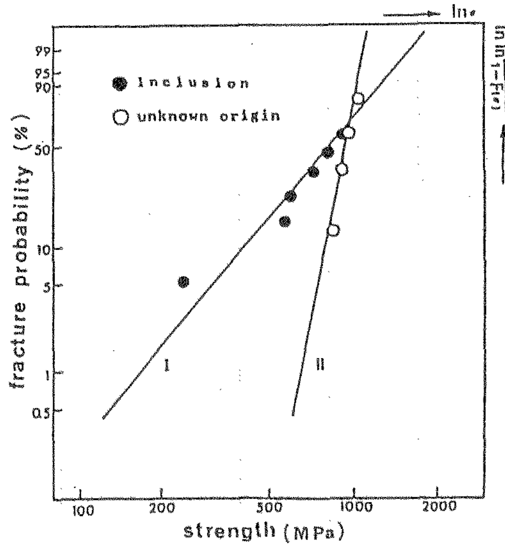


Fig.5 Weibull plots for fracture origins. Fracture probabilities are determined by the Johnson method. Straight lines I for inclusion and II for unknown origin are given by F_{T1} and F_{T2}, respectively.

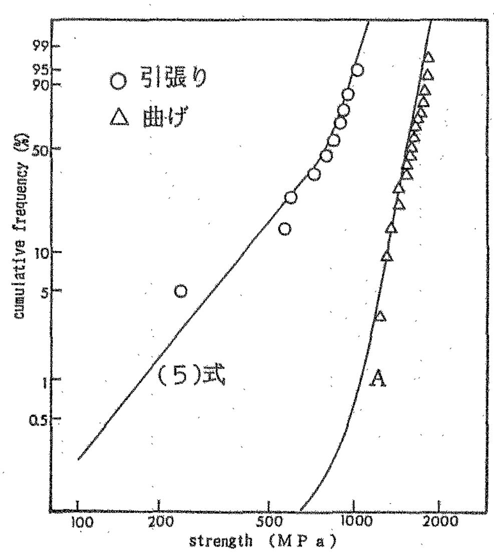


Fig.6 Weibull plots for sample A. Distribution curves I for tensile strength and II for three-point bend strength are calculated from Eqs.(6) and (8), respectively.

Table Ⅱ. Tensile and Three-Point Bend Strengths of 2Y-TZP

	Tensile			Three-point bend		
Sample	Mean (MPa)	CV+ (%)	n*	Mean (MPa)	CV (%)	n
A	745	29	10	1630	10	17

+ CV is coefficient of variation. * n is number of tested specimens.

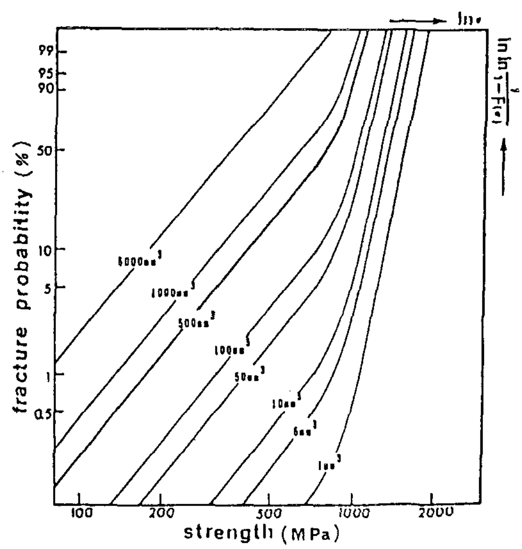


Fig.7 Distribution curves calculated from Eq.(7) for different effective volumes of sample A. Thick line represents the distribution of tensile strenght tested at the volume of 500 mm³ in the present work.