Breakdown Properties of Random Systems with Distributed Conductances

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Abstract

The breakdown problem in a random system is investigated by numerical simulation for a network of distributed conductance . The breakdown voltages are studied in the different conductance configurations. The mean breakdown strength shows anomalous size dependence given by $\langle v_b \rangle \propto 1/((\ln L))^y$ for L (the linear dimension of the network). The exponent y depends upon a degree of non-uniformity of the system and gives an information on the critical event of the breakdown. For the case of a comparatively homogeneous network the micro-crack nucleation is the critical event of a breakdown. In such a random resistor network funnel defects act as the appropriate critical defects. On the other hand, in the case of strongly disordered system the critical event in a breakdown is attributed to the growth of cracks.

1.Introduction

When an externally applied force is increased, a solid breaks into several pieces. This phenomenon is called a fracture. Failure in a stressed system is of great importance in science and technology, and it has received a considerable attention [1]. Motion of a crack tip involves rupturing of only a small number of atomic bonds at a given instant. Hence, the influence of atomic bonds in a local region surrounding the crack tip seems to be an important factor(local effect) .

On the other hand, the non-locality can also play an important role through stress concentration induced by crack or inhomogeneity [2,3] . Machta and Guyer have found that in a random system defects ofa funnel shape give a dominant stress concentration and called them funnel defect [4,5].

Though the aspects of frature is complicated, we believe that some possible universal principles should underly the fracture phenomena. We can imagine various principles , of which the most fundamental one is the fact that the weakest part fails at first. We call this weakest spot in a system as a defect. It is clear that the defects are primarily important in all fracture process and often a few critical defects can determine the fracture strength of the entire system.

Usually, we call a defect a crack when the missing bonds form a line. As configuration of such a crack leads to the amplification of the external stress, cracks existing in the initial flawed configuration are likely to induce the propagation of a fracture. It is well known that the weakest points are produced around edges of the largest crack, because the stress cencentration is proportional to the square root of the length of a crack. When the defect concentrations increase, the weakest point is not always determined by the largest defect but depends also on shielding or enhancement effects which are to be expected when the defect concentration increases. The shielding effect may be regarded as a non-local effect. This shielding effect is well known in continuous mechanics where a crack stops the propagation by adding a dislocation at the location of the crack tip . Such interactions between defects will be essential to the way a solid disrupts and to the dynamics of rupture [6].

The aim of this paper is to study fracture in a stressed disorder system by numerical simulations and hence to clarify some basic concepts which could be useful for the construction of a comprehensive and general theory of failure phenomena in a disordered medium. Classical example of such a fracture is considered to be the mechanical failure in an elastic network and the electrical breakdown in a resistor network. We focus in this paper on an electrical model initially introduced by de Arcangalis et al. [7] , which consists of resistive fuses randomly located on each bond of a lattice. Particularly, the critical types of defects have been studied in the context of fuse networks [8~11] in which the electric breaking originates from the simple microscopic process of the failure of a single bond when the voltage drop across it exceeds a threshold value.

The breakdown of a random network consisting of these elements as the external potential raised is meant to mimic the mechanical fracture of a random elastic network under the condition of increasing uniform tension. But we must notice that the random fuse network method is the scalar analog to the vector problem of the mechanical fracture. The electric potential fulfills the Laplace equation. On the other hand, when we want to take into account the elastic effect we must consider Lame's equation. But we believe that there is qualitatively similar behaviour between these two models for the fracture. Furthermore, we may hope the model of the scalar type to develop general insights about fracture in real materials.

For a random network consisting of nearly identical fuses, the behavior of the system is simple, because the failure of one bond leads to the formation of a linear crack that breaks the entire network. In this case the nucleation is the critical event of the breakdown. But, on looking round an overall physical picture of the breakdown, we find the nucleation to be one aspect of the general problem of the breakdown of an initially crack-free systems. Generally, it is of vital importance to ascertain what is the critical event in the breakdown. If micro-cracks are formed and there is a potential barrier for growth of crack, crack growth process is also the critical event in the breakdown. In this case the breaking process is dominated by the largest crack because of the length-dependent amplification of the current at the crack tip as pointed out by Duxbury et al.[8,9,11]

As pointed out above, there are two critical processes for the breakdown, i.e., the nucleation of micro-cracks and the growth of cracks. In a random system these two processes interplay. In this paper we would like to report our investigation about the size dependence of the average breakdown strength of a fuse network in which the various resistive fuses are randomly located on each bond of a network. By using the results of the average breakdown strength we intend to clarify the role of funnel defects in a disordered system and the breakdown mechanism of a fuse network.

2. Network Simulations.

We consider a random network of fuses. The conductance of the fuses is randomly distributed over the range $\sigma_{min}=1-(w/2)$ to $\sigma_{max}=1+(w/2)$, where σ_{min} is the minimum conductivity and σ_{max} the maximum conductivity [11]. In this simulation the width w shows a degree of the non-uniformity of a system. Each fuse has the breaking point of 1 [A] $(1/\sigma$[V]). Above 1 [A], a fuse becomes an insulator (Fig.1). Now place such fuses at random on the bonds of a two-dimensional lattice. In such a random network we have an inhomogeneous distribution of voltage and current, which in turn controls the damage process. We intend to clarify this phenomenon by means of computer simulation.

We have performed numerical simulation on LxL random fuse networks on the square lattice with L. The length L extends from 10 to 40. The voltage was applied across the horizontal bus bars on opposite edges of the lattice. A free-boundary condition was used for the other two edges (Fig.2). If a sufficiently small voltage is applied then the system conducts just as a random resistor network. When the externally applied voltage becomes large, some of the fuses will be broken. And if enough unmber of the fuses break, the bus bars will no longer be connected, so we will have a breakdown of the entire network.

In this paper we are engaged in the breakdown strength. The breakdown strength of a particular configuration is the lowest externally applied voltage $\langle v_b \rangle$ at which the network breaks down

One way of calculating the breakdown voltage in the fuse network is to perform the following, two-step process iteratively for each lattice configuration. (1) Solve Kirchhoff's equations. We use the conjugate-gradient method to solve Kirchhoff's equations on this netwrok [10]. (2) Find the fuse which is broken.

3. Breakdown Strength and its Size Effect.

In this section we calcualte numerically the breakdown strength as a function of size L of the circuit and then investigate what mechanism dominates in the breakdown process. For the sufficiently small width w , that is nearly homogeneous network, once one broken bond nucleates, there is a tendency that the breaking of the first bond immediately leads to catastrophic failure of the network by the growth of a straight-line crack. Figure 3 displays the configuration of a 20x20 lattice for the width w=0.50 . Bonds that break during the simulation are indicated by blanks. Numeral denotes a magnitude of current passing a bond. In this case at first the micro-crack nucleates at the weakest bond and then propagates through the entire sample in the direction normal to the applied load without further increase in voltage. The external voltage drop V keeps constant during the increase in the number of broken bonds. Hence, the nucleation of a single broken bond determines the entire fracture.

Duxbury et al. [8,9,11] have argued that in the case of weak disorder the breaking process is dominated by the largest crack in the initial state of the system. At the tip of this largest crack, the local current flow is enhanced by a factor which is proportional to the square root of the crack length in two dimensions . Figure 3 shows the current concentration at the crack tip. They have reported that for a system of linear dimension L the length of the largest crack is proportional to ln L. We can explain this result as follows. We imagine a horizontal line defect cluster as a crack . In the L^2 network the number of places in the volume that the line defect cluster can be placed on the lattice is approximately equal to L^2. Let us assume a fraction (or concentration) of the bonds occupied to be p. The remaining fraction (1-p) is of vacant bonds to act as insulators. Hence, the probability of n bonds missing is $(1-p)^n$ and the probability P(n) that n adjacent bonds will be missing is about $P(n)=(1-p)^n L^2$. The largest defect cluster is determined by that value of $n=n_c$ for which P(n) is of order 1 or $(1-p)^n L^2=1$, which implies that the most critical defect size $n_c=\ln L$ surely occurs somewhere in the networks.

In a system of linear dimension L, the length of the largest crack is proportional to ln L and the current enhancement at the tip of this critical defect is of order $(\ln L)^{1/2}$, so that this leads to the average breaking potential of the network vanishing asymptotically as $1/(\ln L)^{1/2}$ with increasing L. We have shown the dependence of the average breaking potential of the network $\langle v_b \rangle$ on size L in Fig.4. The graph is plotted $\langle v_b \rangle$ versus ln L on a double logarithmic scale for the case of w=0.24 , which corresponds to the ratio $\sigma_{min}/\sigma_{max}=0.73$. The simulation results lies on the straight line. This result shows that, even we use a system with the free boundary conditions in the transverse directions, there is not a strongly enhanced probability of broken bonds near the free surfaces of the network. From Fig.4 it is clear that $\langle v_b \rangle$ is a decreasing function of $1/(\ln L)^y$ and the exponent y is 0.0425.

This result leading to the too small exponent y is contradict to Duxbury et al.'s argument that the breaking process is controlled by the largest crack in the system. In their result the exponent y is proposed to be about 1/2. Our results reflects the fact that in a comparatively homogeneous medium the nucleation of micro-cracks rather than the growth of cracks is the critical event for the breakdown. In a crack-free system there should be defects which may concentrate the current only a bit necessary to nucleate micro-cracks.

Machta and Guyer [4,5] have proposed funnel-type defects to distribute in a random circuit. The funnel defect gives rise to the current concentration. They have studied the random resistive network with two different nonzero conductivities and proposed that the funnel defect was the domain defect in this problem. A funnel defect funnels a current proportional to its size ℓ of the defect cluster through the most critical bond (for example, in two dimensions two horizontal cracks with a few bonds between them have this property. See the bonds indicated by the arrow in Fig.8) . The current density reachs a finite maximum in the critical bond. The maximum current increases as a power law in (ℓ/a) where a is lattice constant. As the system size increases, larger defects (configurations) appear and the maximum current in the network increases. They have obtained the logarithmic size dependence of the breakdown strength and reported the logarithmic size dependence of the breaking strength to be a robust feature of breakdown in a variety of models, though the exponent y may be dependent upon the field (voltage etc.) and on the breakdown law of a single bond. They also have shown the enhancement exponent y is dependent on the ratio of the two conductivities, i.e., $\sigma_{min}/\sigma_{max}$. The factor y ranges from 1/2 to 0 according $\sigma_{min}/\sigma_{max}$ changing from 0 to 1.

We show the results for the cases of w=0.6 and 1.0 in Figs.5 and 6. The exponent y =0.0913 for w=0.6 and y=0.214 for w=1.0, respectively. We have also plotted the dependence of the exponent y on the ratio $\sigma_{min}/\sigma_{max}$ in Fig.7. Our numerical results are shown by the circles, and the dashed line is the analytical results of Machta and Guyer. In the range of the small width w, our results are consistent with Machta's analytical results. In this range the funnel model is more appropriate to the random resistor circuits as compared with the crack model.

For larger the width w, the breakdown of the network is more gradual than in the brittle fracture of small w. Figure 8 shows the case of the width w=2.0. The sample does not fail instantenuously when the first bond breaks. Further increase in the applied voltage leads to additional non-catastrophic failures . In this case the final failure path, that is the critical crack, is far from straight and there is considerable damage to the network .

As seen from Fig.7, in the case of the large w, i.e., w=0.6 ($\sigma_{min}/\sigma_{max}$=0.54), there is apparently small difference between our results and Machta' results. In such a more random state defects begin interacting and larger defects or cluster appear. In the limit of w=2 our results rather approaches to Duxbury's result [12]. In the limit w 2 Duxbury et al. predict that the exponent y goes to y=1/(D-1) where D is the spatial dimension. Hence, in the present case of D=2 the exponent y approaches to y=1. This consequence seems an aspect that there is limitation of the validity of the two components funnel model. This limitation reflects appearance of new fracture process, that is the critical defect to be a crack. Machta et al. have talked that there is a dicontinuity in y at w=2 and that the discontinuity is a crossover from the dominance of funnel to linear critical defect.

4. Hot Spots and Distribution of Breakdown Strengths.

The calculation of the previous section have been for the average breakdown strength in the breakdown process. We have said that in the case of a crack-free system initiation of the breakdown process has been attributed to resistors with high current. Usually regions where the density of the dissipated power is substantially larger than in the surrounding areas are called hot spots. Machta et al. have called the resistor distributions giving to those hot spots as "funnel defects". This critical defect appears as a result of long-range effects of the resistor distributions. In order to test the hypothesis that the funnel defect is a critical defect, we study the spatial distribution of current in a system. Figures 9(a,b,c) show the spatial fluctuation of current in the case of w=1.5. Figure 9(a) shows that there is the current concentration at the (2,11) beside the left free boundary. After increasing the voltage a bit there occurs a different distribution of the current and there appears another funnel defect at (11,7) (see Fig.9(b)) . Our simulation further confirms that hot spots are results of strongly nonlocal properties of the resistor distribution. The constriction of the current to a few narrow necks plays an important role in the evolution of hot spots. The hot spots grow parallel to the applied electric field.

Figure 9(c) shows this point to be broken and there to nucleate the broken bond. Helsing et al. [13] have shown how the field in a single resistor depend on the resistor itself (local effect) and on its surrounding networks (long-range effect). They also have reported that the long-range effect in two dimensions, related to the resistor distribution surrounding on a hot spot has a characteristic double-cone shape with opening angle of 90° and cone axis along the applied field. The well-conducting links form connected paths which are narrowed at the hot spot due to the blocking by poorly-conducting links. Our results are consistent to Helsing's conclusions.

The existence of hot spots is not determined only by the resistor distribution in the immediate vicinity of the hot spot. Instead, it also strongly depends upon the distribution of the resistors in a large region surronding the particular spot. Hence, hot spots should be regarded as results of global, rather than local properties of the phase distribution.

In the case of the breakdown strength, it is possible to calculate the form of the full distribution of breakdwon strengths. In other word, it is to consider the probability that a network has failed at a voltage v or less. The calculation relies upon the same hypothsis that the eventual failure of the network is dominated by the most critical defect in the network. The calculation of the full distribution function of breakwon strengths then reduces to the calculation of the distribution functions of these most critical defects. In order to know new breakdown process, we have investigated the voltage dependence of the number of fractures. The data represents 500 trials on a network.

The distribution of $\langle v_b \rangle$ is a relatively sensitive probe of the underlying breaking mechanism. For small w , in comparatively pure limit the distribution function of number of the breaking strengths resembles Gaussian distribution (see Fig.10). This calculation results relies upon the hypothesis that there is only type of breakdown process and that the eventual failure of the network is dominated by the nucleation of a micro-crack in the network. In the region of the lower side of the distribution to be exponential in v, the cumulative distribution for failure voltage F(v) is expressed in the Weibull form. In order to test for conformity to a Weibull distriution ,$F(v) = 1- \exp(-cL^2v^m)$ where c and m are constants. We plot $\ln[(1-F(v))]$ against $-[(v-10)/15]^3$. The agreement between the simulation results and the Weibull distribution is good (see Fig.11).

On the other hand, for the larger w, the main peak itself decreases and shifts to the lower voltage side . The distribution broadens and displays several peaks on lower side of the main peak (Fig.12). When the randomness starts incresing, some defects are close enough to interact and finite clusters of missing bonds are present and then a large crack develops. Finally, the maximum disturbance in the current distributions is created by long thin defect orientated perpendicular to the average current flow. And the crack propagates across the lattice of already present structures.

5.Conclusion.

We have studied the fracture in a random resistor network by numerical simulations. The breaking voltage in the network has been studied as a function of the size. For a weakly disordered network, numerical simulations are in agreement with Machta et al.'s theoretical prediction, that is the funnel defect to be the critical defect. The breakdown of a strongly disordered network is governed by the growth of cracks as pointed out by Duxbury and shows the tensile properties rather than the brittle properties in the case of weakly disordered system. For intermediate case, there are complicated interactions between defects.

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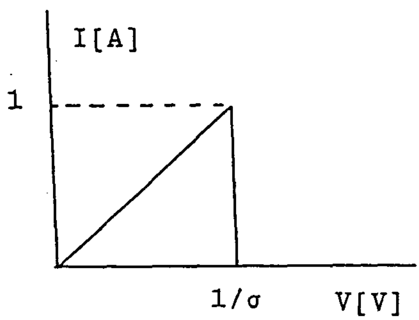


Fig.1. I-V characteristic for a single fuse. Above a current of 1[A], no current flows.

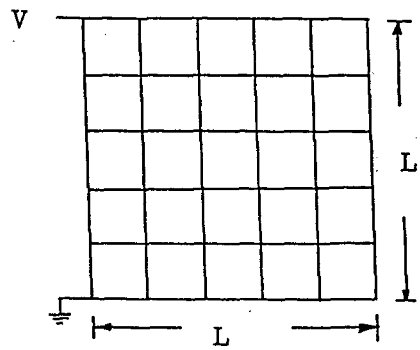


Fig.2. Network for a L x L square lattice.

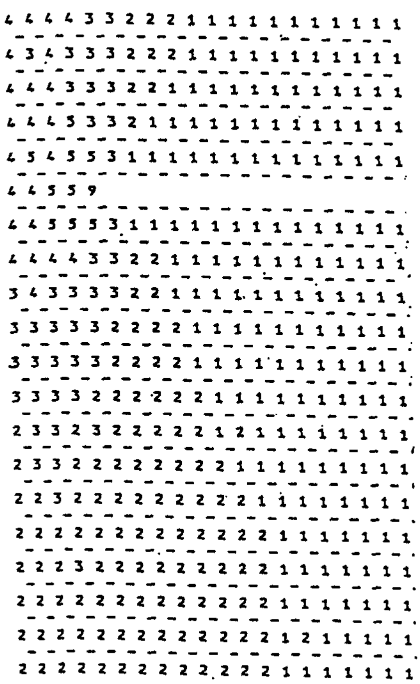


Fig.3. Snapshot of a network for the case of w=0.5. Numeral denotes a magnitude of current passing a bond. The numeral '1' indicates the current 0.91[A] and the numeral '9' the current 0.99[A].

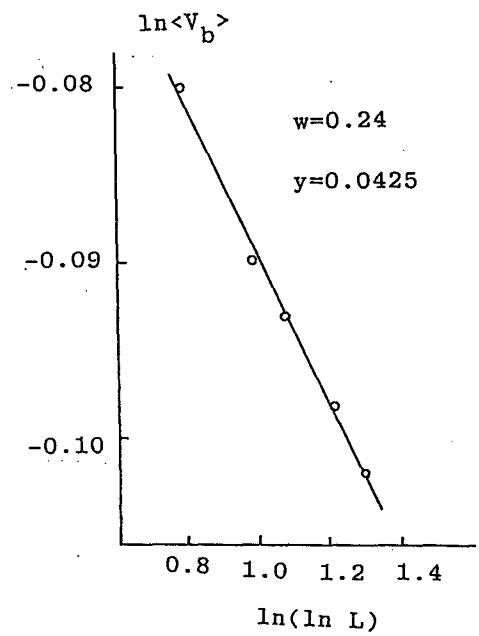


Fig.4. Plot of $\ln\langle V_b\rangle$ vs ln(ln L) for the case of w=0.24. The slope of this plot gives -1/y.

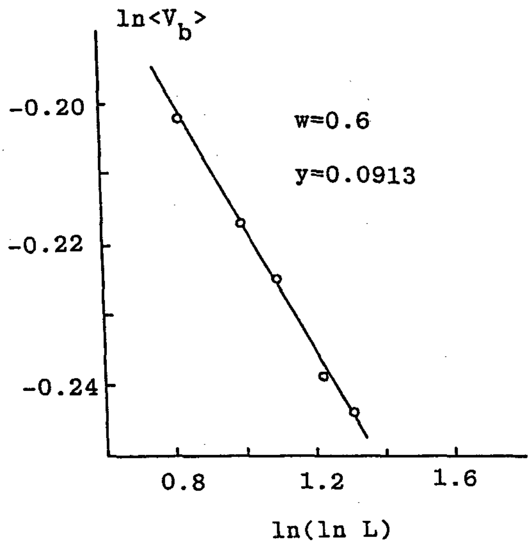


Fig.5. Plot of $\ln\langle V_b\rangle$ vs ln(ln L) for the case of w=0.6

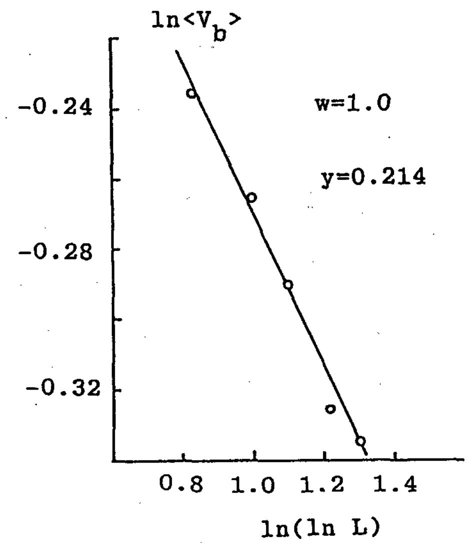


Fig.6. Plot of $\ln\langle V_b\rangle$ vs ln(ln L) for the case of w=1.0.

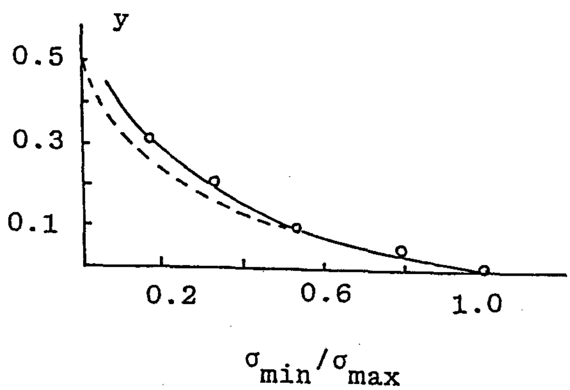


Fig.7. The exponent y vs $\sigma_{min}/\sigma_{max}$. The circles are our results and the dashed line is Machta's funnel model.

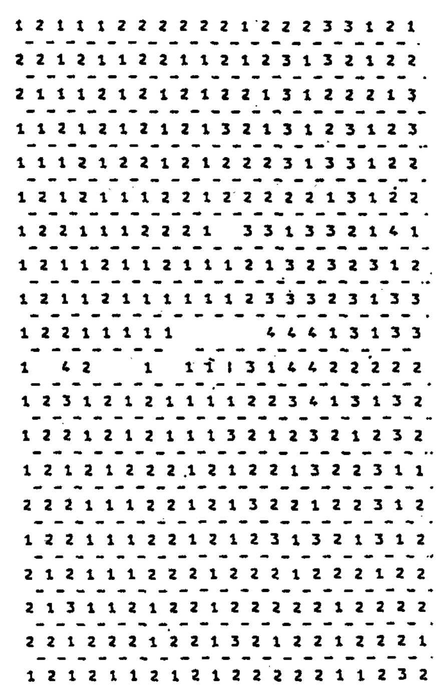
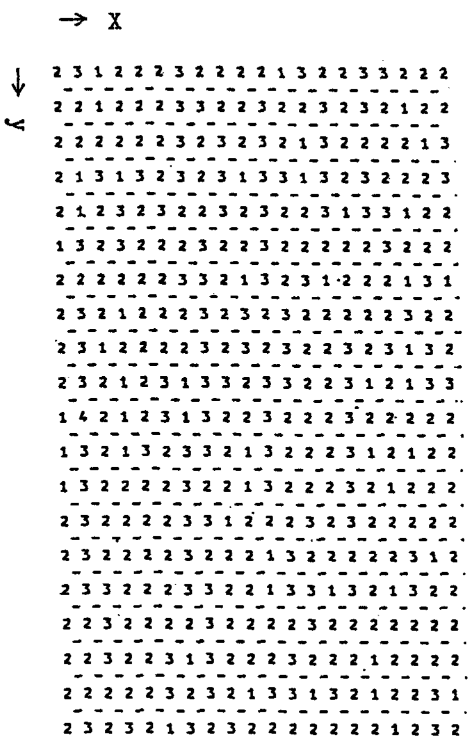
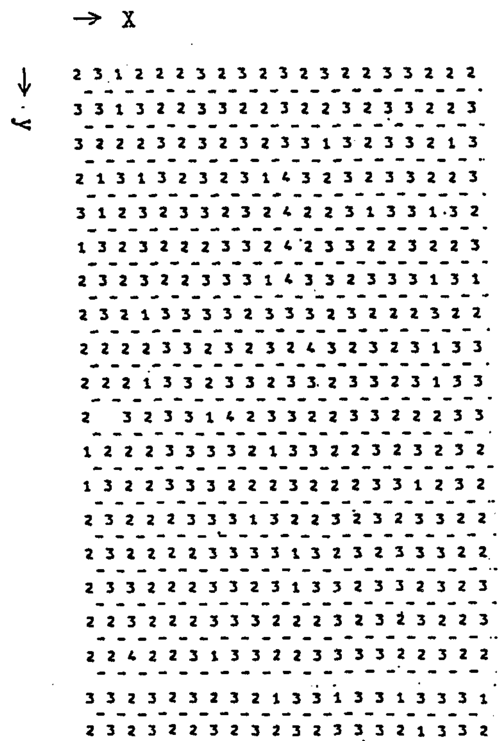


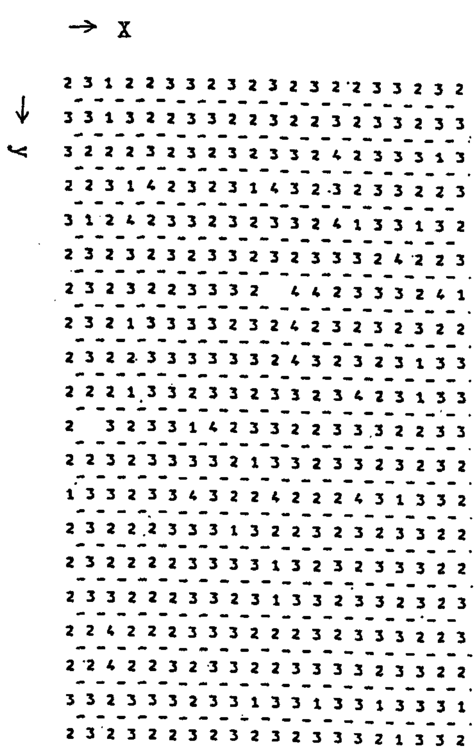
Fig.8. Snapshots of a network for the case of w=2.0. We indicate the hottest bond in the network by the arrow.



(a)



(b)



(c)

Fig.9. Snapshots of a network for the case of w=1.5.
(a) The position vector of a hot spot is (2,11).
(b) and (c) The position vectors of two hot spots are (2,11) and (11,7).

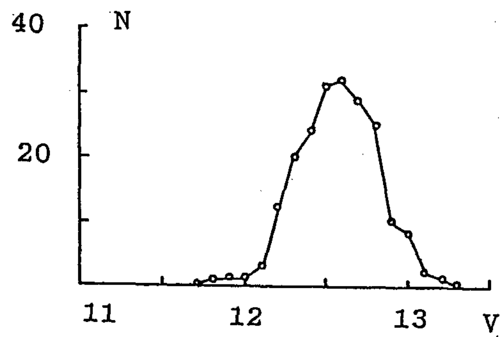


Fig.10. The distribution of breaking strengths vs applied voltage V for the case of w=0.5.

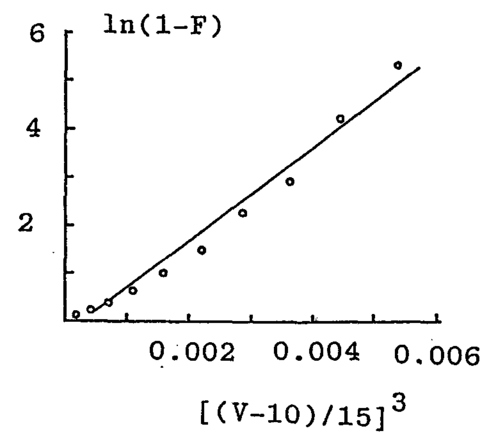


Fig.11. The cumulative distribution of the breaking strengths vs applied voltage V for the case of w=0.5.

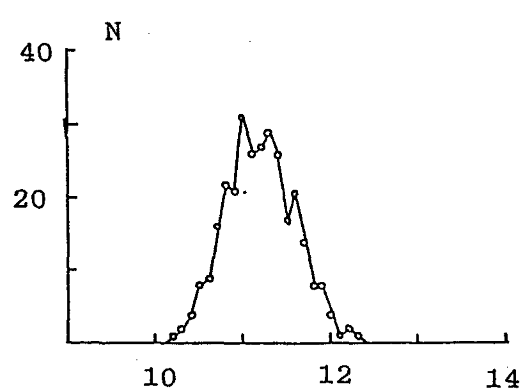


Fig.12. Number of breaking strengths vs applied voltage V for the case of w=2.0.