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Numerical Study on the Vortex State for Bose-Einstein Condensations in Two-components Gas-Phases of Alkali-Metal Atoms

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Characteristic features in phases of Bose-Einstein condensations (BEC) for systems of two-components in gas phases of Alkali-metal atoms trapped by harmonic potentials are discussed on the basis of numerical calculations by the use of Gross-Pitaevskii equation. Condensations with and without vortices are considered in order to make clear spatial distributions. We emphasize the possibility of coexistence of vortex-free state (where the angular momentum ℓ is 0.) for the one component in BEC and vortex state ($\ell_z = 1$) for another component. Furthermore, the dynamics of expanding condensates after releasing the magnetic trap. In particular, we concentrate our attention to different points of the behavior in the expanding process between for the condensate with a vortex and that without vortex.

KEYWORDS: Bose-Einstein condensation, gases of Alkali-metal atoms, two-components including a vortex, Gross-Pitaevskii equation, the dynamics of expanding condensates after releasing the trap

§1. Introduction

The recent observations of Bose-Einstein condensation (BEC) has enabled a series of beautiful experiments that probe the dynamics of the macroscopic quantum states described by the macroscopic wavefunctions.[1-4] Furthermore, two-components for spin states of (F=2,m=2) and (F=1,m=-1) are taken into account by the hyperfine interaction as for ^{87}Rb. In recent a few years, in fact, we have several papers[5-8] as to condensations of BEC for systems of two-components. When the two-components are overlapped, additional novel features are expected[7,8] to appear. In these situations for the investigation of these gases, a detailed theoretical understanding of models for multi-components is needed.[9] In particular, the investigation should be made from the viewpoint of the difference between cases with and without vortices. Such results can be discussed by theoretical calculation on the basis of Gross-Pitaevskii equation (GPE). In this paper, we discuss, therefore, properties of BEC in Alkali atom gases of two-components for cases with and without vortices by the use of GPE. Furthermore, we discuss the dynamics of the expanding condensates after the releasing magnetic trap which is described as the the harmonic potential. In particular, we concentrate our attention to different features of the behavior between for the condensate with a vortex and that without vortex.

§2. GP equation

In order to discuss the behavior of BEC for two-component systems, we make the consistent numerical calculation for this non-linear equation of GPE.

From the mathematical point of view, GPE in the present model is the nonlinear coupled Schrödinger equation for two-components 1 and 2. In fact, we obtain the following coupled equations between $i = 1, 2$:

$$\mu_i\psi_i(\vec{r}) = \frac{-\hbar^2}{2M}\nabla^2\psi_i + u_{ii}N|\psi_i(\vec{r})|^2\psi_i(\vec{r}) + u_{ij}N|\psi_j(\vec{r})|^2\psi_i(\vec{r}) + \frac{1}{2}M\omega^2 r^2\psi_i(\vec{r}), \quad (1)$$

where M is the mass of the atom. Here, the condensed wavefunction ψ_i per atom is normalized as

$$\int |\psi_i|^2 d^3\vec{r} \quad = 1 \quad (2)$$

The magnetic trap is described as a harmonic-oscillator type with the angular frequency ω. The interaction strength u_{ij} is expressed by the s-wave scattering length a_{ij} as follows:

$$u_{ij} = 4\pi\hbar^2 a_{ij}/M. \quad (3)$$

where μ_i is the chemical potential for i component determined consistently.

If we consider the case where the cylindrical rotation with the verocity $\vec{v}_\theta = (\hbar/M)\nabla_\theta$, we can express $\vec{v}_\theta$ as $(\hbar/\rho M) < \ell_z >$ by the angular momentumm $< \ell_z >$ of this rotation. Here, we use $\nabla_\theta = (1/\rho)(\partial/\partial\theta)$, in which we adopt the cylindrical coordinates (ρ, θ, z). Taking into account the centrifugal potential caused by the angular momentumm ℓ_z to GPE (1), we can get the state with the vortex for $< \ell_z >$.

§3. Method of Numerical Calculation of Gross-Pitaevskii Equation for States with a Vortex

We make the consistent calculation for GPE in the expression (1). In the numerical calculation, we adopt units of energy and length are $\hbar\omega$ and $d_0 = (\hbar/M\omega)^{1/2}$. We rewrite the wavefunction ψ_i in GPE(1) as the following form in order to express the state with vortex by the cylindrical coordinates:

$$\psi_i = \{g_i(\rho_0, z_0)exp(i\ell_{iz}\theta)\}/(\sqrt{2\pi}\cdot d_0^{3/2}), \quad (4)$$

in which $\rho_0 = \rho/d_0$ and $z_0 = z/d_0$. Accordingly, GPE turns into the equation for $g_i(\rho_0, z_0)$ as

$$\begin{aligned}
\beta_1 g_1 = & -\frac{1}{2}\partial^2 g_1/\partial\rho_0^2 - \frac{1}{2}(1/\rho_0)\partial g_1/\partial\rho_0 \\
& -\frac{1}{2}\partial^2 g_1/\partial z_0^2 \\
& +\frac{1}{2}(\ell_{1z}^2 g_1/\rho_0^2) + \alpha_{11}|g_1|^2 g_1 + \alpha_{12}|g_2|^2 g_1 \\
& +\frac{1}{2}(\rho_0^2 + z_0^2)g_1 \\
\beta_2 g_2 = & -\frac{1}{2}\partial^2 g_2/\partial\rho_0^2 \\
& -\frac{1}{2}(1/\rho_0)\partial g_2/\partial\rho_0 - \frac{1}{2}\partial^2 g_2/\partial z_0^2 \\
& +\frac{1}{2}(\ell_{2z}^2 g_2/\rho_0^2) - \alpha_{22}|g_2|^2 g_2 + \alpha_{12}|g_1|^2 g_2 \\
& +\frac{1}{2}(\rho_0^2 + z_0^2)g_2, \qquad (5)
\end{aligned}$$

Giving the value of ℓ_{iz} to GPE (5), we can obtain states with and without a vortex: In the present work, we treat cases of ($\ell_{1z} = 0, \ell_{2z} = 0$) and ($\ell_{1z} = 0, \ell_{2z} = 1$).

Here,

$$\alpha_{ij} = 4\pi N a_{ij}/d_0. \quad (6)$$

The spatial extension R^3 of the present BEC can be estimated to be $R = \alpha_{11}^{1/5}d_0$, because the potential energy $(1/2)M\omega^2R^2$ in the trap corresponds to the interaction energy between atoms $u_{11}N/R^3$.

Now, we introduce the parameter V as

$$V^2 = \frac{\alpha_{12}/\alpha_{11}}{\alpha_{22}/\alpha_{11}}, \quad (7)$$

which is independent of N.

§4. Numerical Results for Static Seate for Macroscopic Condensates

In the numerical calculation, we adopt $a_{11} = 100a_B$ and $a_{22} = 200a_B$ (where a_B is Bohr radius),$N = 5340$ and $\omega = 2\pi \times 220H_z$. Thus, d_0 is $7.24 \times 10^{-7}m$ for the case of Rb atoms. By the use of these values, strengths of interaction of eq.(6) α_{11} and α_{22} in GPE are determined to be 1000 and 2000, respectively. It should be noted that the value of V defined to be (7) becomes 0.707 at $\alpha_{12} = \alpha_{11}$ for the present case of $\alpha_{22} = 2\alpha_{11}$.

We show the spatial distribution for the static states in Fig.1(a)(b), in which wavefunctions $g(\rho_0)$ vs ρ_0 of 1- and 2- components are drown for various values of the parameter V. Because of $\psi_2(0) = 0$ the donut-like distribution appears for any value of V.

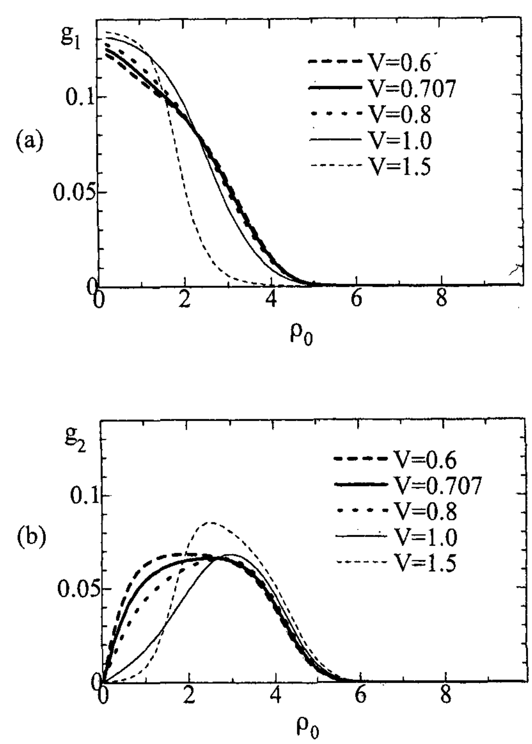

Fig. 1. The spatial distribution of wavefunctions in the case where only 2-component has the vortex ($\ell_{2z} = 1$). The wavefunctions of 1- and 2- components are drown for various values of the parameter V. Because of $g_2(0) = 0$ the donut-like distribution of 2-component appears for any value of V.

§5. Dynamics of Condensed State for Releasing the Trap

We proceed our discussion to the problem of how the wavefunctions behave after the releasing the trap. We treat such the dynamics for cases in which the condensate Ψ_1 has a vortex for $\ell_{1z} = 1$, while Ψ_2 remains the condensate without vortex, i.e. $\ell_{2z}=0$. This situation can be more unstable than that discussed for the static state in the previous section. In calculation, we replace μ_i in GPE (5) by $i \cdot \frac{\partial}{\partial t}$. In addition, the boundary condition of rigid walls at $\rho_0 = 10.2$ and $z = -10.2, 10.2$ is applied.

Numerical results for the dynamics are shown in Fig.2 and Fig.3 in which the expanding to the radial direction with increasing time are illustrated for the vortex state (a) and vortex-free state (b).

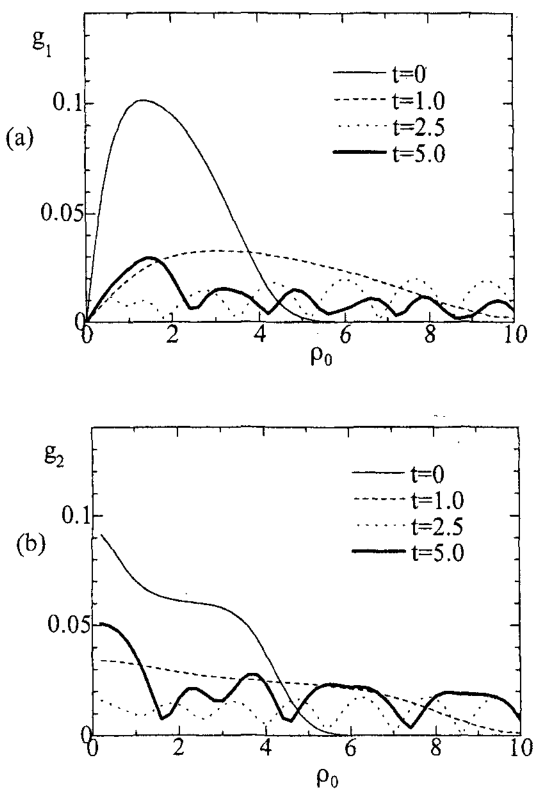

Fig. 2. The behavior of condensed states for the expanding to the radial direction after releasing the trap. (a) The state with a vortex. (b) The state without vortex.

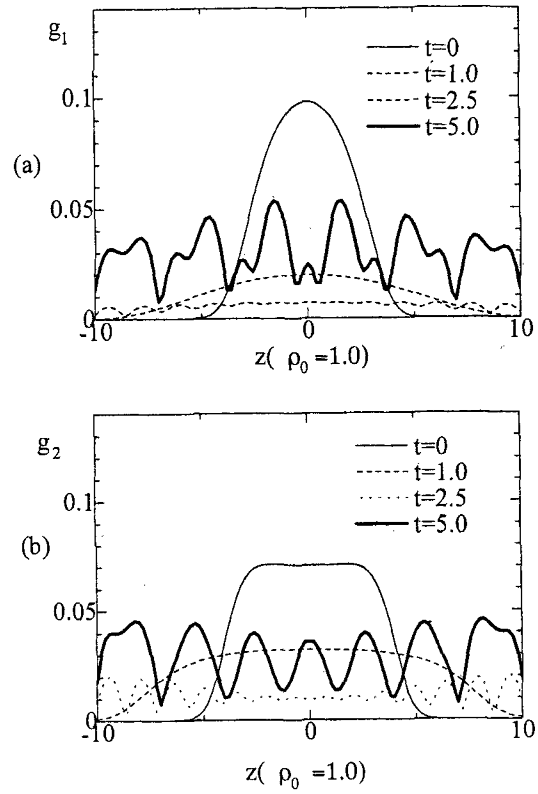

Fig. 3. The behavior of condensed states for the expanding to the z direction after releasing the trap. (a) The state with a vortex. (b) The state without vortex.

§6. Summary

In the present paper, we show the characteristic features in phases of BEC for systems of two-components of Alkali gases trapped by harmonic potentials on the basis of numerical calculations for GPE. In partiqular, we discuss the case in which one component includes a vortex and another one has no vortex. The spatial structure of these coexistence is made clear.

Furthermore, we show the expanding those condensates after the releasing the trap. The behavior of the expanding condenstates is made clear in relation with the effect of the vortex which is included in the one component. We would like to expect that this work provides a clue to understanding features of BEC in two-component systems and the dynamics of those condensations in connection with the problem of the essential role of the vortex.

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