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Behavior of Piezoelectric Ceramics under High Sinusoidal Vibration Stress

Yasuhiro Sasaki, Sadayuki Takahashi*, Mikio Umeda**, Kentaro Nakamura*** and Sadayuki Ueha***

NEC Corporation, 4-1-1, Miyamae-ku, Kawasaki-shi, 216-8555, Japan
FAX:81-44-856-8885, e-mail:y-sasaki@ds.jp.nec.com
*Waseda University, 3-4-1 Shinjuku-ku, Okubo Tokyo 169-8555, Japan
FAX: 81-3-3232-3578, e-mail: sa-takahashi@h8.dion.ne.jp
**Nagaoka National College of Technology, 888 Nishikatakai-cho, Nagaoka, Niigata 940-8532, Japan
FAX:81-258-34-9700, e-mail:umeda@nagaoka-ct.ac.jp
***Tokyo Institute of Technology, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan
FAX:81-45-924-5961, e-mail:knakamur@pi.titech.ac.jp
FAX:81-45-924-5961, e-mail:sueha@pi.titech.ac.jp

ABSTRACT

Electromechanical characteristics of piezoelectric ceramics (PZT) under a resonant mode with a high level of vibration stress were theoretically and experimentally studied. Though the vibration is nonlinear (non-sinusoidal), it is found that the behavior of the vibration can be approximated by the fundamental harmonic vibration (sinusoidal). Elastic compliance s_{11}^E and mechanical loss factor Q_m^{-1} are quadratic function of vibration stress for hard PZT and a linear function for soft PZT, respectively. Piezoelectric constant d_{31}, however, is a linear function of vibration stress both for hard and soft PZTs.

Key words: piezoelectric ceramics, sinusoidal, stress, nonlinear, harmonic

1. INTRODUCTION

Piezoelectric responses of ferroelectric ceramics consist of intrinsic and extrinsic parts [1]. The intrinsic part refers to the lattice deformation within every individual ferroelectric domain. The extrinsic part represents the elastic deformation which largely originates from the motions of non-180° domain walls [2]. In many ferroelectric piezoelectric ceramics, such as barium titanate $BaTiO_3$ (BT) and lead-zirconate-titanate $Pb(Zr,Ti)O_3$ (PZT), the extrinsic effect contributes to 60-70% of the experimentally observed piezomoduli [3]. The electromechanical characteristics of piezoelectric ceramics show nonlinear behavior when the ceramic is subjected to a strong electric field or high stress [4-6]. They are functions of both electric field and stress [7]. It is believed that irreversible wall motions occurring above this threshold cause the significant nonlinearity.

In recent years, intensive development efforts have been devoted to new high-power piezoelectric ceramic devices, such as ultrasonic motors and piezoelectric transformers. Piezoelectric ceramics used in these devices are usually driven with a large vibration amplitude in a resonant mode. It is important to understand the above-mentioned nonlinear behavior which appears in the relatively high-power or large amplitude range. However, this effect has not yet been thoroughly studied.

In this report, the nonlinear effect of piezoelectric ceramics is discussed both theoretically and experimentally.

As specimens for this study, we employed soft and hard PZT transducers. The transducers were driven in a longitudinal mode of piezoelectric transverse-effect under electric field-free (E=0) boundary condition. The measurement was carried out using the electrical transient response method [8].

2. THEORETICAL ANALYSIS OF NONLINEAR EFFECT

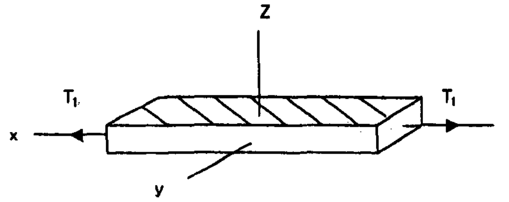
We used the piezoelectric ceramic transducer illustrated in Fig. 1 for this analysis. The transducer, which is polarized in the thickness direction, has dimensions of length l, width b and thickness t. For the mechanical resonance with electric field E=0, the strain S_1 and electric flux D_3 are expressed by a function of stress T_1 as follows [9]:

$$S_1 = s_{11}^E T_1 + s_{111}^E T_1^2 + s_{1111}^E T_1^3 + \cdots \quad , \tag{1}$$

$$D_1 = d_{31} T_1 + d_{311} T_1^2 + d_{3111} T_1^3 + \cdots \quad , \tag{2}$$

$$T_1 = T_0 \sin(\omega_0 t)\cos\left(\frac{\pi}{l}x\right) \quad , \qquad (3)$$

where T_0 and ω_0 are the amplitude of stress and the angular resonant frequency, respectively. The s_{11}^E, s_{111}^E, and s_{1111}^E are the 2nd, the 3rd, and the 4th elastic compliance, d_{31}, d_{311} and d_{3111} are the 2nd, the 3rd and the 4th piezoelectric constants, respectively.



$l \times b \times t$

Fig. 1. Piezoelectric ceramic transducer polarized in the thickness direction.

When the spectral analysis are applied to formulas (1) and (2), each harmonic without the DC term is represented by formulas (4) to (9).

$$S_1^{(1)} = \left(s_{11}^E + \frac{3}{4}s_{1111}^E T_0^2 \cos^2\left(\frac{\pi}{\ell}x\right) + \ldots\right)$$
$$\times T_0 \sin(\omega_0 t)\cos\left(\frac{\pi}{\ell}x\right), \qquad (4)$$

$$S_1^{(2)} = \left(\frac{1}{2}s_{111}^E T_0 \cos\left(\frac{\pi}{l}x\right) + \cdots\right)$$
$$\times T_0 \sin\left(2\omega_0 t - \frac{\pi}{2}\right)\cos\left(\frac{\pi}{l}x\right), \qquad (5)$$

$$S_1^{(3)} = \left(\frac{1}{4}s_{1111}^E T_0^2 \cos^2\left(\frac{\pi}{l}x\right) + \cdots\right)$$
$$\times T_0 \sin(3\omega_0 t - \pi)\cos\left(\frac{\pi}{l}x\right), \qquad (6)$$

$$D_3^{(1)} = \left(d_{31} + \frac{3}{4}d_{3111}T_0^2 \cos^2\left(\frac{\pi}{l}x\right) + \cdots\right)$$
$$\times T_0 \sin(\omega_0 t)\cos\left(\frac{\pi}{l}x\right), \qquad (7)$$

$$D_3^{(2)} = \left(\frac{1}{2}d_{311}T_0 \cos\left(\frac{\pi}{l}x\right) + \cdots\right)$$
$$\times T_0 \sin\left(2\omega_0 t - \frac{\pi}{2}\right)\cos\left(\frac{\pi}{l}x\right), \qquad (8)$$

$$D_3^{(3)} = \left(\frac{1}{4}d_{3111}T_0^2 \cos^2\left(\frac{\pi}{l}x\right) + \cdots\right)$$
$$\times T_0 \sin(3\omega_0 t - \pi)\cos\left(\frac{\pi}{l}x\right). \qquad (9)$$

In the case that we can neglect the 2nd and higher order harmonic terms, the amplitudes of strain S_0 and electric flux D_0 are respectively represented by formulas (10) and (11).

$$S_0 \cong S_0^{(1)} \cong s_{11}^E\left(1 + \alpha_1 T_0^2\right)T_0, \quad \alpha_1 = \frac{3s_{1111}^E}{4s_{11}^E}, \qquad (10)$$

$$D_0 \cong D_0^{(1)} \cong d_{31}\left(1 + \beta_1 T_0^2\right)T_0, \quad \beta_1 = \frac{3d_{3111}}{4d_{31}}. \qquad (11)$$

If we express the current amplitude of each harmonic as $i_0^{(1)}$, $i_0^{(2)}$ and $i_0^{(3)}$,

$$i_0^{(1)} = b\omega_0 \int_{-\frac{l}{2}}^{\frac{l}{2}} D_3^{(1)}dx \cong b\omega_0 d_{31}T_0 \int_{-\frac{l}{2}}^{\frac{l}{2}} \cos\left(\frac{\pi}{l}x\right)dx = \frac{2}{\pi}lb\omega_0 d_{31}T_0 \quad , \qquad (12)$$

$$i_0^{(2)} = 2b\omega_0 \int_{-\frac{l}{2}}^{\frac{l}{2}} D_3^{(2)}dx \cong b\omega_0 d_{311}T_0^2 \int_{-\frac{l}{2}}^{\frac{l}{2}} \cos^2\left(\frac{\pi}{l}x\right)dx = \frac{1}{2}lb\omega_0 d_{311}T_0^2 \propto \left[i_0^{(1)}\right]^2 \quad , \qquad (13)$$

$$i_0^{(3)} = 3b\omega_0 \int_{-\frac{l}{2}}^{\frac{l}{2}} D_3^{(3)}dx \cong \frac{3}{4}b\omega_0 d_{3111}T_0^3 \int_{-\frac{l}{2}}^{\frac{l}{2}} \cos^3\left(\frac{\pi}{l}x\right)dx = \frac{1}{\pi}lb\omega_0 d_{3111}T_0^3 \propto \left[i_0^{(1)}\right]^3 \qquad (14)$$

When the vibration velocity amplitude of each harmonic on the edge of the transducer ($\pm x = l/2$) is expressed as $v_0^{(1)}$, $v_0^{(2)}$ and $v_0^{(3)}$,

$$v_0^{(1)} = \omega_0 \int_0^{\frac{l}{2}} S_1^{(1)}dx \cong \frac{1}{\pi}l\omega_0 s_{11}^E T_0 \quad , \qquad (15)$$

$$v_0^{(2)} = 2\omega_0 \int_0^{\frac{l}{2}} S_1^{(2)}dx \cong \frac{1}{4}l\omega_0 s_{111}^E T_0^2 \propto \left[v_0^{(1)}\right]^2, \qquad (16)$$

$$v_0^{(3)} = 3\omega_0 \int_0^{\frac{l}{2}} S_1^{(3)}dx \cong \frac{1}{2\pi}l\omega_0 s_{1111}^E T_0^3 \propto \left[v_0^{(1)}\right]^3. \qquad (17)$$

While, the force factor of each harmonic $A_0^{(1)}$, $A_0^{(2)}$, and $A_0^{(3)}$ can be expressed using the formulas (12) to (17).

$$A^{(1)} = \frac{i_0^{(1)}}{v_0^{(1)}} \cong \frac{2b d_{31}}{s_{11}^E} \quad , \qquad (18)$$

$$A^{(2)} = \frac{i_0^{(2)}}{v_0^{(2)}} \cong \frac{2b d_{311}}{s_{111}^E} \quad , \qquad (19)$$

$$A^{(3)} = \frac{i_0^{(3)}}{v_0^{(3)}} \cong \frac{2b d_{3111}}{s_{1111}^E} \quad . \qquad (20)$$

In above theoretical formulas, we assume that there is no hysteresis in the behaviors in S(T) and D(T). If the hysteresis exists, then,

$$S_0 \cong S_0^{(1)} \cong s_{11}^E(1+\alpha_2 T_0)T_0 \quad , \qquad (21)$$

$$D_0 \cong D_0^{(1)} \cong d_{31}(1+\beta_2 T_0)T_0 \quad , \qquad (22)$$

where α_2 and β_2 are coefficients, respectively [7].

3. EXPERIMENTAL

The measurement was carried out using the electrical transient response method [8]. Five kinds of lead-zirconate-titanate (PZT) ceramics were used as transducer material: N10, N21, N6, N8, and N81 (Tokin Products). The N10 and N21 are typical soft PZT, N6 and N8 are typical hard PZT, and N81 is a hard PZT newly developed for use in high-power piezoelectric transformers [10].

These transducers, which are polarized in the thickness direction, operate in a longitudinal vibration mode of the piezoelectric transverse-effect.

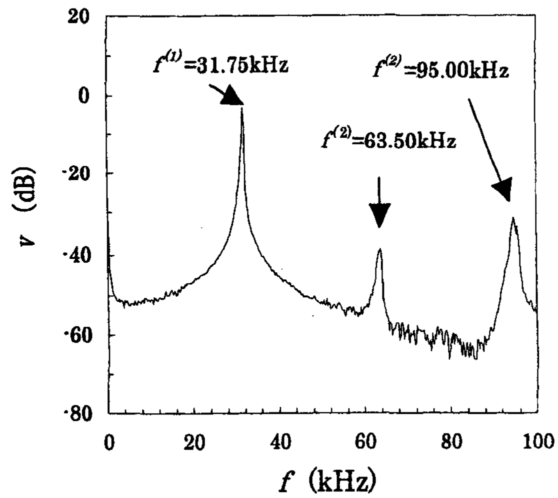


Fig. 2. Spectrum of vibration velocity v.

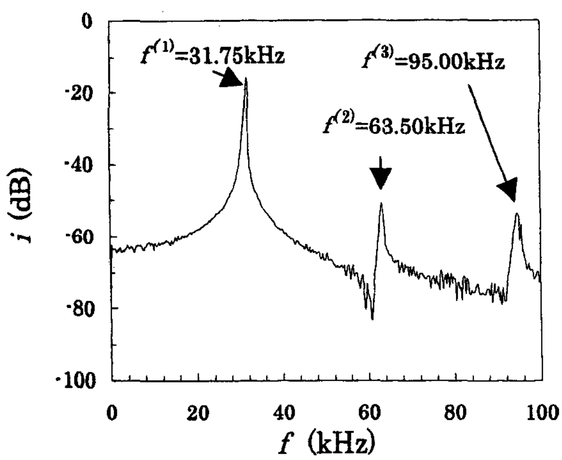


Fig. 3. Spectrum of current i.

Table I Electromechanical properties at low vibration level

Ceramics	N10	N21	N6	N8	N81
Density ρ ($\times 10^3$ kg/m^3)	7.95	7.82	7.92	7.93	8.01
Elastic compliance s_{11}^E ($\times 10^{-12}$ m^2/N)	14.8	16.3	12.7	11.8	8.7
Dielectric constant ε_{33}^T	5440	1880	1450	1010	480
Piezoelectric constant d_{31} ($\times 10^{-12}$ C/N)	-278	-198	-133	-122	-59
Mechanical quality factor Q_m	70	63	1400	1450	1650

Table I lists density ρ, elastic compliance s_{11}^E, dielectric constant ε_{33}^T, piezoelectric constant d_{31}, and mechanical quality factor Q_m of each material. Electromechanical characteristics were evaluated at a low level of vibration stress.

3.1 Spectral analysis [11]

The frequency spectra for the vibration velocity v and current i were analyzed using the transducer composed of N8 with l=52 mm, b=5 mm, and t=1 mm.

The frequency spectra of v and i just after the electrical terminals are shunted are shown in Figs. 2 and 3. Three peaks are observed at the frequencies $f^{(1)}$=31.75 kHz, $f^{(2)}(2f^{(1)})$=63.5 kHz, and $f^{(3)}(3f^{(1)})$=95 kHz. These spectra indicate that the vibration is not linear but nonlinear. The peaks at $f^{(1)}$, $f^{(2)}$, and $f^{(3)}$ are, respectively, the fundamental, the 2nd, and the 3rd harmonic vibration due to nonlinear effects.

Figures 4 and 5 represent the amplitudes of vibration velocity in the 2nd harmonic $v_0^{(2)}$ and the 3rd harmonic $v_0^{(3)}$ as a function of the fundamental harmonic $v_0^{(1)}$. Since the values of $v_0^{(2)}$ and $v_0^{(3)}$ are much smaller than that of $v_0^{(1)}$, the 2nd and higher harmonics are approximately negligible. The relations $v_0^{(2)} \propto [v_0^{(1)}]^2$ and $v_0^{(3)} \propto [v_0^{(1)}]^3$ can also be seen. These results give fairly good agreement with the theoretical formulas (16) and (17).

The amplitudes of current in each harmonic $i_0^{(1)}$, $i_0^{(2)}$, and $i_0^{(3)}$ are plotted against $v_0^{(1)}$, $v_0^{(2)}$, and $v_0^{(3)}$ in Figs. 6, 7, and 8, respectively. The results $i_0^{(1)} \propto v_0^{(1)}$, $i_0^{(2)} \propto v_0^{(2)}$, and $i_0^{(3)} \propto v_0^{(3)}$ are also in accord with the theoretical formulas (18), (19), and (20).

3.2 Dependence of electromechanical characteristics on vibration stress

In this section, we focus on the fundamental harmonic because the higher order terms are negligible.

We evaluated the dependence of elastic compliance s_{11}^E, piezoelectric constant d_{31}, and mechanical loss factor Q_m^{-1} on vibration stress. The transducers with l=42 mm, b=5 mm, and t=1.0 mm were used in this experiment.

The fractional changes of $\Delta s_{11}^E/s_{11(0)}^E$, $\Delta d_{31}/d_{31(0)}$, $\Delta Q_m^{-1}/Q_{m(0)}^{-1}$ against maximum amplitude of vibration stress T_0 are respectively shown in Figs. 9, 10, and 11. Here, the $s_{11(0)}^E$, $d_{31(0)}$, and $Q_{m(0)}^{-1}$ are the values measured at a low level of vibration. The results for the hard PZT give empirical formulas which show the same proportion like formula (10).

$$s_{11}^E \cong s_{11(0)}^E\left(1+\alpha_s T_0^2\right) , \qquad (23)$$

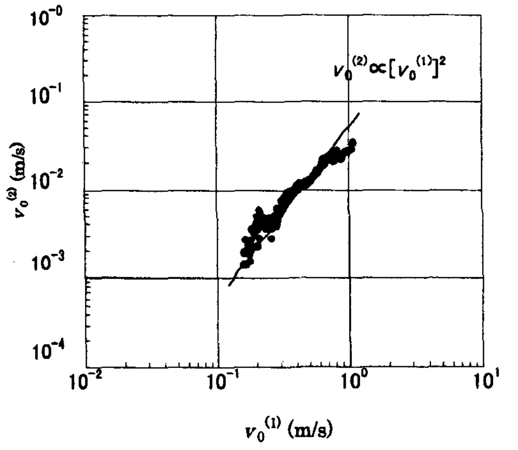


Fig. 4. Amplitude of vibration velocity in the 2nd harmonics $v_0^{(2)}$ vs. amplitude of vibration velocity in the fundamental harmonics $v_0^{(1)}$.

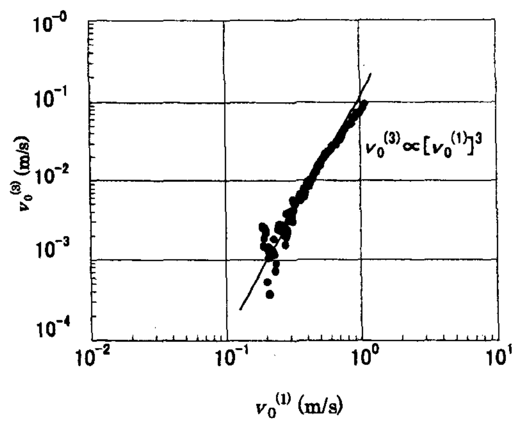


Fig. 5. Amplitude of vibration velocity in the 3rd harmonics $v_0^{(3)}$ vs. the amplitude of vibration velocity in the fundamental harmonics $v_0^{(1)}$.

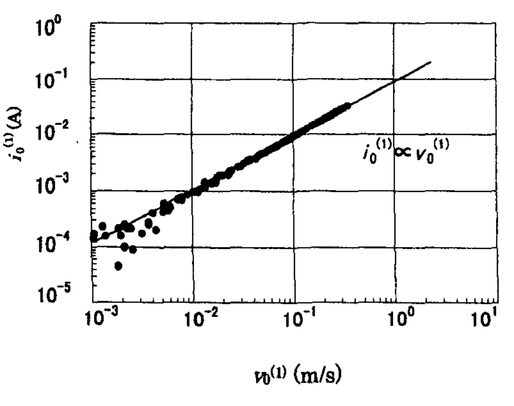


Fig. 6. Amplitude of current in the fundamental $i_0^{(1)}$ vs. amplitude of vibration velocity in the fundamental harmonics $v_0^{(1)}$.

$$Q_m^{-1} \cong Q_{m(0)}^{-1}\left(1+\alpha_Q T_0^2\right) \ . \qquad (24)$$

This suggests that the S(T) for hard PZT behaves with the small hysteresis. The empirical formulas for soft PZT are obtained as follows:

$$s_{11}^E \cong s_{11(0)}^E\left(1+\alpha_S' \, T_0\right), \qquad (25)$$

$$Q_m^{-1} \cong Q_{m(0)}^{-1}\left(1+\alpha_Q' \, T_0\right). \qquad (26)$$

These formulas are in good agreement with the theoretical formula (21). This explains the reason that S(T) for the soft PZT shows large hysteresis.

On the other hand, d_{31} for both soft and hard PZT is represented by the following of formula,

$$d_{31} \cong d_{31(0)}\left(1+\alpha_d T_0\right) \ . \qquad (27)$$

This result suggests that D(T) for both soft and hard PZT has large hysteresis.

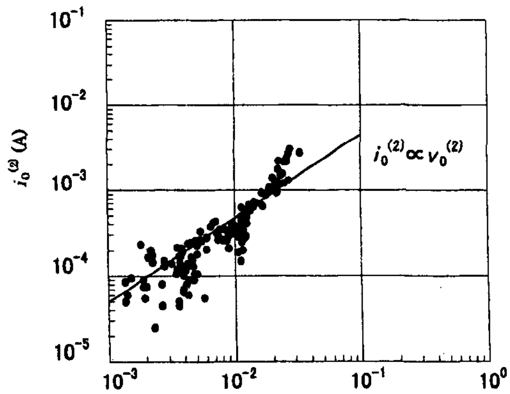


Fig. 7. Amplitude of current in the 2nd $i_0^{(2)}$ vs. amplitude of vibration velocity in the 2nd harmonic $v_0^{(2)}$.

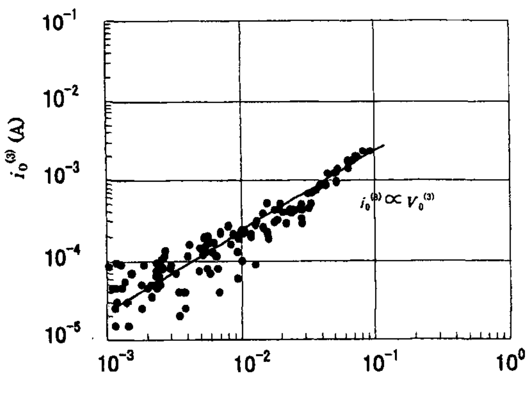


Fig. 8. Amplitude of current in the 3rd $i_0^{(3)}$ vs. amplitude of vibration velocity in the 3rd harmonic $v_0^{(3)}$.

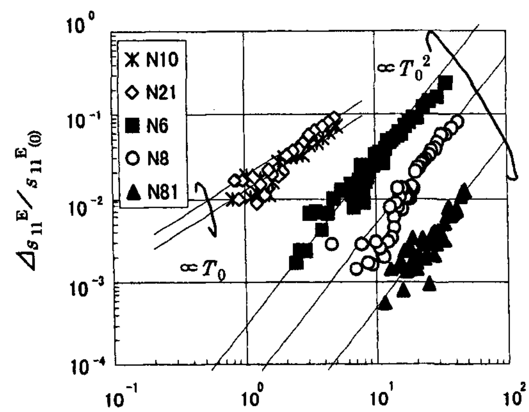


Fig. 9. Fractional change in elastic compliance $\Delta s_{11}^E / s_{11(0)}^E$ vs. amplitude of vibration stress T_0.

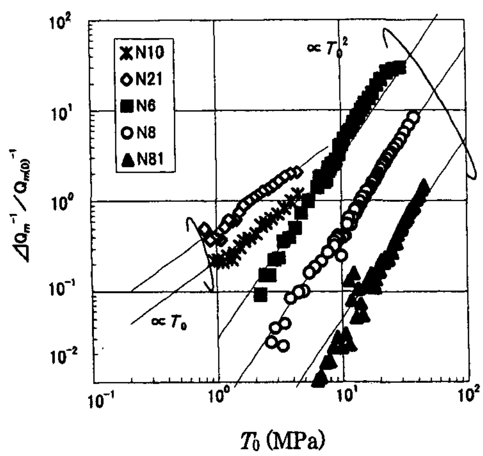


Fig.10. Fractional change in mechanical loss factor $\Delta Q_m^{-1}/Q_{m(0)}^{-1}$ vs. amplitude of vibration stress T_0.

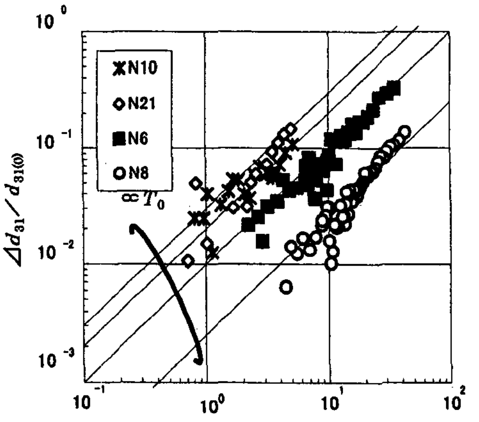


Fig.11. Fractional change in piezoelectric constant $\Delta d_{31}/d_{31(0)}$ of vs. amplitude of vibration stress T_0.

4. SUMMARY

Nonlinear behavior of PZT, operating in the longitudinal mode of piezoelectric transverse-effect under an electric field-free (E=0) boundary condition, was evaluated from both sides of the theory and from experimental results. The results obtained clearly show that the elastic compliance s_{11}^E and the mechanical loss factor Q_m^{-1} are quadratic functions of vibration stress T_0 for hard PZT, and that the piezoelectric constant d_{31} is a linear function of T_0, where s_{11}^E, Q_m^{-1}, and d_{31} for the soft PZT are approximated by linear functions of T_0.

In light of experimental results and theoretical formulas produced to date, a further important point is that S(T) and D(T) for soft PZT have hysteresis, while, the hysterisis of S(T) is negligible but that of D(T) is not negligible for hard PZT.

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