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DRIVING CONDITIONS OF NONLINEAR PHENOMENA IN PIEZOELECTRIC CERAMICS

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Using *h*-form nonlinear piezoelectric equation translated from the *g*-form equation, the driving condition, in which current-jump phenomena appear, was studied in the piezoelectric ceramic vibrators with a rectangular shape. The jump phenomena are observed when the vibrators are driven at the frequency lower than a critical frequency. As far as the driving frequency is lower than the critical frequency, the driving voltage inducing the jump decreases with the increase of driving frequency. The current jump easily occurs in high Q_m materials since the higher Q_m value, the lower the inducing voltage.
key words: piezoelectric ceramics, high-power driving, current jump phenomenon, nonlinear piezoelectric equation, mechanical quality factor

1. INTRODUCTION

We have studied the current jump phenomena observed frequently in the piezoelectric ceramics driven at around the resonance frequencies.[1,2] The current-jump phenomenon is one of the typical nonlinear phenomena, and is a main factor disturbing the stability of the high-power running.[3] The generation mechanism and the driving conditions of the jump phenomena have not been made clear enough yet. In this paper, the driving condition inducing the jump phenomena and the relationship between the driving condition and the mechanical quality factor Q_m of the ceramics are reported.

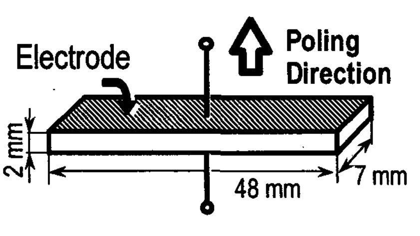

Fig. 1. Geometry of the piezoelectric ceramic vibrator.

2. EXPERIMENTAL

The sample configuration was a rectangular bar of size $48\times7\times2$ mm^3 as shown in Fig. 1. A pair of silver electrodes was fired on the upper and lower surfaces of the rectangular bar, and the sample was poled under an electric field of 3 kV/mm for 10 min in silicone oil bath kept at 120 ℃. The sample is "hard material" of Q_m = 1700 and was prepared using a conventional method detailed in a previous paper [4]. The samples were driven at around the fundamental resonance frequency of length extensional 1/2 λ mode. The constants of the equivalent circuit were measured using an LF impedance analyzer (HP, 4192A) with small signal field. The displacements were detected by a laser vibrometer (Graphtec, AT0042). The measurements were done by the constant-voltage circuit described in a previous paper [1,2].

3. RESULTS AND DISCUSSION

Figure 2 shows the typical current response to the frequency in current-jump phenomena. When the driving voltage is higher than a threshold voltage, the shape of current spectrum is unsymmetric, and hysteresis is observed as displayed by open circles in the figure. When the frequency is increased from the lower frequency side to higher frequency side, the magnitude of the current traces the curve of E→A→B →C. When the frequency is decreased from the C point, the current changes along the curve of C→B→D →E. From point A and from point D the current jumps and drops down, respectively. These behaviors of current were analyzed by means of the piezoelectric equation.

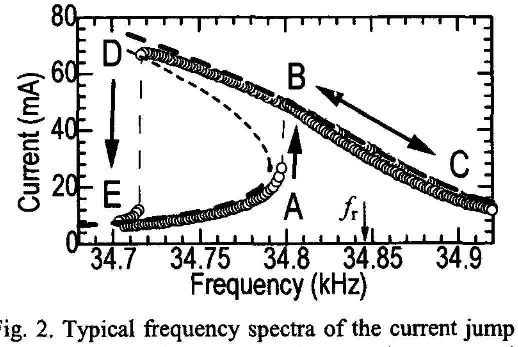

Fig. 2. Typical frequency spectra of the current jump phenomenon. The open circles are the measured plots. Theoretical broken curve of the current-jump phenomenon was obtained using Eq. (4).

There are four forms, *d*-form, *g*-form, *e*-form and

h-form, in piezoelectric equation. When a rectangular piezoelectric vibrator shown in Fig. 1 was driven by a sinusoidal current having the resonance frequency of length-extensional $1/2\,\lambda$ mode vibration, the boundary conditions are as follows; $E_1=E_2=0$ and $E_3\neq 0$ for electric fields, $D_1=D_2=0$ and $D_3\neq 0$ for electric flux density, $T_1\neq 0$ and $T_2=T_3=0$ for stress, $S_1\neq 0$, $S_2\neq 0$ and $S_3\neq 0$ for strain. When these boundary conditions are strictly satisfied, the most widely used and easily treated equation is the d-form piezoelectric equation because it does not include S as an independent variable. Using the nonlinear piezoelectric equation based on the d-form equation, the nonlinearity of piezoelectric ceramics has been frequently studied.[5] However, the analysis is very complex since d-form equation has T as an independent variable which we can not detect easily. It is probably difficult that the nonlinear phenomena represented with the current jump are quantitatively analyzed using d-form equation.

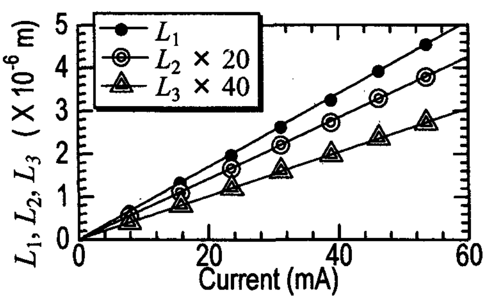

Fig. 3. Relation of amplitudes between current and displacement in the direction of length, L_1, width, L_2, and thickness, L_3.

On the other hand, the h-form equation has S and D as independent variables which we can easily measure. By introducing the circuit equation obtained by starting from the h-form equation, the quantitative analysis can be effectively done. In order to satisfy the boundary conditions, the h-form equation translated from the g-form equation is employed for the analysis as shown in Eq. (1). Here, Fig. 3 shows the relationship of the amplitudes between the current and displacement in the direction of the length, width and thickness of the sample driven by the constant-current method in which the current jump doesn't appear.[4] The result indicates that exact linear relationships exist among D_3, S_1, S_2 and S_3 at around the resonance frequency even when the current is larger than 50 mA in which the jump phenomena should appear in the constant-voltage circuit. Therefore, the nonlinear terms can be expressed with an only variable D_3 by utilizing the proportional relationships among D_3, S_1, S_2 and S_3.

$$E_3 = -\frac{g_{31}}{s_{11}^{D}}S_1 + \beta_{33}{}^{S}D_3 + \gamma_{D31}D_3{}^2 + \xi_{D31}D_3{}^3 \qquad (1)$$

The γ_{D31} and ξ_{D31} are intrinsic nonlinear constants as material constants. It is assumed that the relationship among linear terms receives no influence even if nonlinear phenomena appear, and that nonlinear voltages generated in the sample can be simply added to the voltage of linear terms. This is one of the approximations by perturbation, which is applied when nonlinear terms are small compared with linear terms. Equation (2) is obtained by superposing nonlinear terms on the solution of linear terms at around the resonance frequency.

$$v_3 = L\frac{di_3}{dt} + Ri_3 + \frac{1}{C}\int i_3dt + \gamma_{D31}'\left(\int i_3dt\right)^2 + \xi_{D31}'\left(\int i_3dt\right)^3 \qquad (2)$$

Here, L, C, R are inductance, capacitance, and resonance resistance of the equivalent circuit. The γ_{D31}' and ξ_{D31}' are effective nonlinear constants for the vibrator.[6] Assuming that the sample current is sinusoidal (primary approximation [7]), the relation between voltage and current for fundamental frequency is obtained from Eq. (2) as Eq. (3). Equation (3) includes the fundamental frequency component generated due to nonlinear coefficient of ξ_{D31}'.

$$v_0 \sin\omega t = (\omega L - 1/\omega C)i_0\cos(\omega t+\theta) + Ri_0\sin(\omega t+\theta) - \frac{3\xi'}{4\omega^3}i_0{}^3\cos(\omega t+\theta),$$
$$\theta = \tan^{-1}\left[\left\{-(\omega L - 1/\omega C) + \frac{3\xi'}{4\omega^3}i_0{}^2\right\}\Big/R\right] \qquad (3)$$

The amplitude of the voltage is represented as Eq. (4).

$$v_0 = \sqrt{\left\{(\omega L - 1/\omega C)i_0 - \frac{3\xi_{D31}'}{4\omega^3}i_0{}^3\right\}^2 + (Ri_0)^2} \qquad (4)$$

The jump phenomenon is theoretically demonstrated using Eq. (4). The values of L, C, and R for the calculations are 140 mH, 149 pF, and 20 Ω, respectively. Since it is difficult to analytically solve Eq. (4) for the current, the ω-i_0 spectra were obtained by graphically solving the i_0-v_0 curves as shown in Fig. 4(a). Figure 4 (a) shows voltage current characteristics with frequency as a parameter. The current values against a voltage are found in the point that the voltage is equal to v_0 at the each frequency in Fig. 4(a), and the current response curve to the frequency was drawn as Fig. 4(b) by plotting the currents derived from Fig. 4(a). In Fig.4(a), ω_A and ω_D are the angular frequencies in the case that the current jumps up and drops down, respectively. The three solves of the current (i_{M1}, i_{M2}, i_{M3}) exist at the angular frequency of ω_M. While i_{M1} is the stable value of current when the driving frequency increases from lower frequency side, the stable value is i_{M3} during the frequency decrease. The broken curves in Fig. 1 shows the theoretical values obtained from Eq. (4). The value of ξ_{D31}' is 5×10^{20} V/C^3 which was estimated from the magnitude of 3rd harmonic voltage measured in the constant-current method[6]. The theoretical curves agree well with the experimental values.

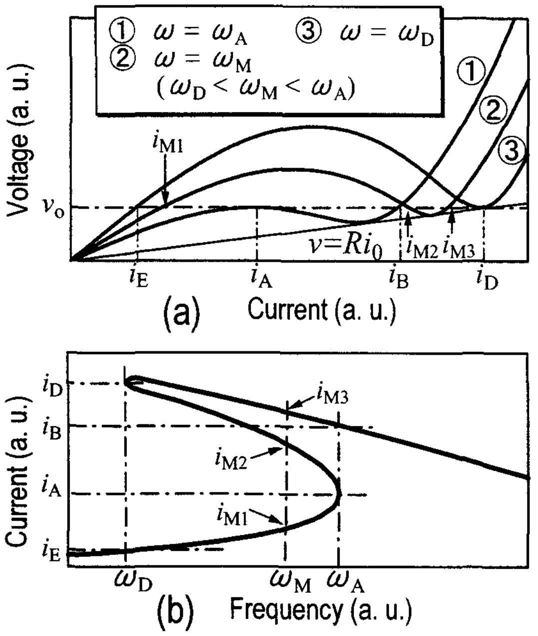

Fig. 4. Graphically solving method of Eq. (4) for a simulation of the jump phenomenon. Current-voltage curve based on Eq. (4) at each frequency, (a); and the current response of frequency obtained from Fig. 4(a), (b).

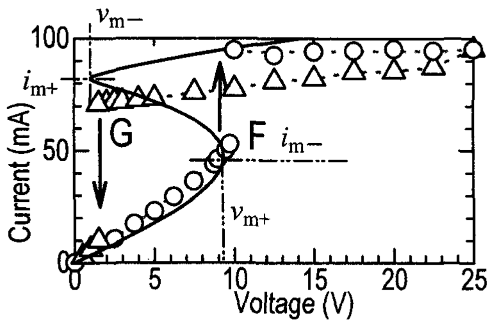

Fig. 5. Voltage response of current measured at the constant frequency of 34.7 kHz, and theoretical curve calculated using Eq. (4). The theoretical curve is described as a bold line. The circle plots show the values measured when the driving voltage is increased, and triangular ones are the values when the voltage is decreased.

Figure (5) shows the v_0-i_0 curve calculated from Eq. (4) with the measurement values at the constant frequency of 34.7 kHz. The current jumps are observed from point F (i_{m-}, v_{m+}) and point G(i_{m+}, v_{m-}). The current jumps up and drops down since the current avoids the unstable negative-resistance region from point F to point G. When the negative-resistance region exists in the theoretical v_0-i_0 curve, the v_0 shows extremal values against i_0 ($\partial v_0/\partial i_0 = 0$) in Eq. (4). Hence, $i_{m\pm}$ is expressed as Eq. (5).

$$i_{m\pm}^{\ 2} = 4\omega^3/9\xi_{D31}'\left\{2(\omega L - 1/\omega C) \mp \sqrt{(\omega L - 1/\omega C)^2 - 3R^2}\right\} \tag{5}$$

Because $i_{m\pm}$ is real number, the angular frequency ω must satisfy the following condition.

$$(\omega L - 1/\omega C)^2 - 3R^2 \geq 0 \tag{6}$$

As shown in Fig. 1, the jump phenomena appear in the lower frequency region than $f_r = \omega_r/2\pi = (CL)^{-1/2}/2\pi$. The f_c is the resonance frequency when the sample is driven by a low voltage signal. Since the sign of $(\omega L - 1/\omega C)$ is negative in this case, the angular frequency, $\omega_c = 2\pi f_c$, at which Eq. (6) is equal to zero, shows the upper limit for the appearance of the jump phenomena. The jump phenomena never appear when the driving frequency exceeds f_c, even if the driving voltage increases or decreases. Substituting Eq. (5) into Eq. (4), the driving voltage inducing the jump, $v_{m\pm}$, is obtained as shown in Eq. (7).

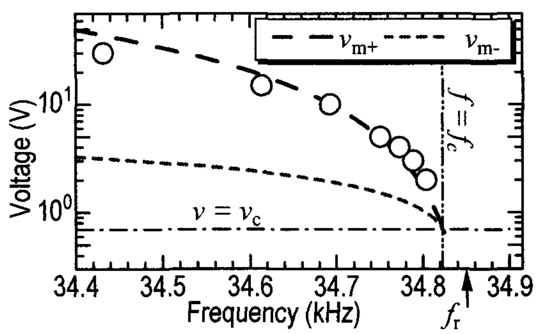

Fig. 6. Measured and calculated driving voltage which induces the current jump at each frequency. The open circles are measured values, and broken line are theoretical curves calculated using Eq. (7).

$$v_{m\pm} = \frac{4\omega^3}{9\xi_{D31}'}\left\{2(\omega L - 1/\omega C) \pm \sqrt{(\omega L - 1/\omega C)^2 - 3R^2}\right\} \times \sqrt{\frac{\xi_{D31}'}{2\omega^3}\left\{(\omega L - 1/\omega C) \mp \sqrt{(\omega L - 1/\omega C)^2 - 3R^2}\right\}} \tag{7}$$

The broken curve in Fig. 6 was calculated from Eq. (7). The open circles illustrate the measured voltage values inducing the current jump from point F at the each frequency. The theoretical curve well represents the actual condition of the jump voltages. The critical voltage v_c, which is v_m value at f_c, indicates the lowest voltage inducing the jump phenomenon in the piezoelectric ceramics. From Eqs. (6) and (7), v_c is expressed with respect to $Q_m = 1/R\sqrt{L/C}$ as Eq. (8).

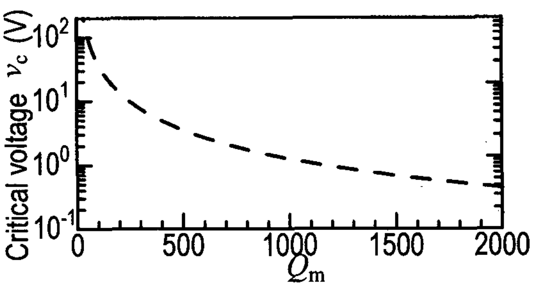

Fig. 7. The effect of Q_m on critical voltage calculated using Eq. (8).

$$v_c = \frac{2\sqrt{6}}{9Q_m^{\,2}C}\left(-\sqrt{3}+\sqrt{3+4Q_m^{\,2}}\right) \times\sqrt{\frac{-\sqrt{3}}{2\xi_{D31}'Q_m^{\,2}C}\left(-\sqrt{3}+\sqrt{3+4Q_m^{\,2}}\right)} \tag{8}$$

Equation (8) suggests that the jump phenomena appear at the lower driving voltage when the larger 3rd harmonic voltage is generated in the sample, since v_c is proportional to $(-\xi_{D31}')^{-1/2}$. Figure 7 shows the relationship between v_c and Q_m when the values of L and C are fixed and R is variable. The figure theoretically indicates that the v_c decreases with increase of Q_m, that is to say, the current jump is induced with the lower driving voltage. This is consistent with the empirical fact that the jump phenomena are more frequently observed in higher Q_m materials.

4. CONCLUSIONS

From the theoretical calculation based on *h*-form nonlinear piezoelectric equation translated from the *g*-form equation, the following conditions, in which the current jump phenomena appear, were found.

1. As far as the driving frequency is lower than critical frequency, f_c, the driving voltage inducing the jump, v_m, decreases with increase in the driving frequency. The lowest v_c which can induce the jump is equal to v_m value at f_c.
2. The v_c is proportional to $(-\xi_{D31}')^{-1/2}$
3. The higher Q_m, the smaller v_c, that is, the jump phenomena occur easily in high Q_m materials.

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