

## In-Phase Motion of Vortices in Intrinsic Josephson Junctions in Mesas of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$ Single Crystals

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We have systematically studied the Josephson vortex flow in stacks of intrinsic Josephson junctions in mesas of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$  single crystals. The current-voltage ( $I$ - $V$ ) characteristics along the  $c$ -axis direction of the mesas were measured at 4.2 K under comparatively low magnetic fields up to 0.2 T perpendicular to the  $c$ -axis. The vortex-flow branches were observed in addition to a superconducting and multiple quasiparticle branches on the  $I$ - $V$  characteristics, as a function of magnetic field. It has been found that the vortex flow occurs in only  $\sim 3$  junctions in any mesa, and its velocity increases from  $\sim 3 \times 10^5$  to  $\sim 5 \times 10^6$  m/s with increasing the total number of junctions in the mesa, but independently of the mesa area (in the present case). The observed vortex flow may be explained by a preferential in-phase motion of vortices in different junctions in the mesa, which corresponds mostly to the highest-velocity mode of electromagnetic waves in the stack, according to an inductive-coupling model based on coupled sine-Gordon equations.

Key words: high- $T_c$  superconductor, intrinsic Josephson junction, Josephson vortex, vortex flow, in-phase motion

### 1. INTRODUCTION

Since the discovery of intrinsic Josephson effects in strongly anisotropic layered high- $T_c$  superconductors such as  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$  by Kleiner *et al.* [1] and Oya *et al.* [2] in 1991, the intrinsic Josephson junctions in high- $T_c$  superconductors have attracted considerable attention both from the point of view of their applications in cryoelectronics and of the fundamental physics.

$\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$ , which is typical of such high- $T_c$  superconductors, consists of the stacking of metallic and superconducting  $\text{CuO}_2$  double layers of 0.3 nm thickness alternating with insulating or semiconducting  $\text{BiO}$  and  $\text{SrO}$  layers of 1.2 nm thickness, having the coherence length along the  $c$ -axis  $\xi_c$  of  $\sim 0.1$  nm, and hence consists of a stack of intrinsic Josephson junctions of 1.5 nm thickness [3,4]. Moreover, it has the London penetration depth along the  $c$ -axis  $\lambda_L$  of  $\sim 150$  nm much larger than the thickness of the superconducting layers [5]. In this case, if a magnetic field is applied parallel to the layers in the stack of intrinsic Josephson junctions, Josephson vortices are induced in the junctions, and a strong inductive coupling between junctions and also an interaction between vortices may occur through screening currents around the vortices in them. And hence, a collective motion of vortices with an interaction along different junctions in the stack can occur, which can also excite a cavity resonance (Fiske resonance) or a Josephson plasma in the stack under an appropriate condition, qualitatively differing from that of vortices in a single junction or an anisotropic type-II superconductor. Another advantage of the intrinsic Josephson junctions is the easy fabrication of devices. Thus, the stack of intrinsic Josephson junctions in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$  is a valuable object for the researches not only on the dynamics of Josephson vortices in it but also on the device applications such as high frequency oscillators, detectors and mixers.

The behavior of Josephson vortices in the stack has been studied experimentally, theoretically and numerically. In the early experimental studies, it was examined for single-crystal mesas (with the length  $L$  between 20 and 500  $\mu\text{m}$ , the width  $W$  between 3 and 50  $\mu\text{m}$ , and the thickness  $T$  between 0.26 and 1.2  $\mu\text{m}$ ) under low magnetic fields  $B$  up to 0.02 T at 77 K by Lee *et al.* [6] and under much higher  $B$  up to 1.5 T between 20 and 70 K by Hechtfischer *et al.* [7], and also for single-crystal whiskers under similar  $B$  at 4.2 K by Latyshev *et al.* [8], and then the velocity of vortex flow  $v_F$  was deduced to be  $\sim 4 \times 10^5$  m/s. Recently, a similar result has also been reported from the observation of Fiske steps for mesas containing 5 intrinsic Josephson junctions under  $B$  less than 0.3 T at 4.2 K by Krasnov *et al.* [9]. These results were understood by a vortex flow with the lowest collective-mode velocity, based on a theoretical model built on coupled sine-Gordon equations [7]. In contrast, in our early observation of the vortex flow with a Fiske resonance for a single-crystal mesa with  $L \times W \times T$  of  $37 \times 30 \times 0.018$   $\mu\text{m}^2$  under low  $B$  up to 0.08 T at 11.3 K, we estimated to be  $v_F = \sim 2 \times 10^6$  m/s [10]. This was interpreted to be due to a vortex flow of the highest velocity mode, based on the theoretical model. Furthermore, by our recent studies [11] for much longer mesas under  $B$  up to 0.2 T at 4.2 and 77 K, it has been observed that at 4.2 K, the vortex flow occurs only in 10-20 % of constituent junctions in each mesa, and then  $v_F$  is in the range of  $(1-4) \times 10^6$  m/s, while at 77 K, it occurs in all junctions in the mesa, and then  $v_F$  becomes lower down to  $3 \times 10^5$  m/s. Thus, it is hoped to investigate systematically the dynamical behavior of Josephson vortices in mesas under comparatively low  $B$  at 4.2 K, to make it clearer.

From this point of view, in this paper we report the vortex flow in mesas of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$  single crystals under  $B$  up to 0.2 T at 4.2 K, together with the results of numerical simulations based on the theoretical model.

## 2. THEORETICAL BACKGROUND

The dynamics of Josephson vortices in a long quasi-one-dimensional Josephson junction with a current bias is described by a perturbed sine-Gordon equation. That for a

$$\begin{pmatrix} \varphi_1'' \\ \varphi_2'' \\ \vdots \\ \varphi_n'' \\ \vdots \\ \varphi_N'' \end{pmatrix} = \begin{pmatrix} 1 & S & & & \\ S & 1 & S & & 0 \\ & & \ddots & & \\ & 0 & S & 1 & S \\ & & & \ddots & \\ S & 1 & & & \ddots \end{pmatrix} \begin{pmatrix} \ddot{\varphi}_1 + \alpha \dot{\varphi}_1 + \sin \varphi_1 - \gamma \\ \ddot{\varphi}_2 + \alpha \dot{\varphi}_2 + \sin \varphi_2 - \gamma \\ \vdots \\ \ddot{\varphi}_n + \alpha \dot{\varphi}_n + \sin \varphi_n - \gamma \\ \vdots \\ \ddot{\varphi}_N + \alpha \dot{\varphi}_N + \sin \varphi_N - \gamma \end{pmatrix}, \quad (1)$$

where  $\varphi_n(x, \tau)$  is the phase difference in the  $n$ -th junction, the spatial coordinate  $x$  and time  $\tau$  are normalized to the Josephson penetration depth  $\lambda_J = [\Phi_0/(2\pi\mu_0 d' J_c)]^{1/2}$  and to the inverse Josephson plasma frequency  $\omega_0^{-1} = [\Phi_0 \epsilon_r \epsilon_0 / (2\pi d J_c)]^{1/2}$ , respectively, with the flux quantum  $\Phi_0$ , the permeability of vacuum  $\mu_0$ , the permittivity of vacuum  $\epsilon_0$ , the relative permittivity  $\epsilon_r$ , the critical current density  $J_c$ , the electrode thickness  $t$ , the barrier thickness  $d$  and the effective magnetic thickness  $d' = d + 2\lambda_L \coth(t/\lambda_L)$ ,  $\alpha$  is the dissipation coefficient,  $\gamma$  is the normalized bias current density, and  $S$  is the coupling parameter defined as

$$c_n = \frac{c_{sw}}{\sqrt{1 + 2S \cos[n\pi/(N+1)]}}, \quad n = 1, 2, \dots, N. \quad (2)$$

These velocities are closely related to the dynamic structures of vortex lattices in the vortex-flow regime. The lowest velocity  $c_N$  characterizes the motion of a triangular lattice (out-of-phase mode) and the highest velocity  $c_1$  characterizes the motion of a rectangular lattice (in-phase mode).

This inductive-coupling model accounted very well for experimental observations of dynamical phenomena of Josephson vortices such as flux flow and Fiske resonance in stacks of low- $T_c$  Josephson junctions. Therefore, we also use this model for the interpretation of the dynamic behavior of Josephson vortices observed in the stacks of intrinsic Josephson junctions in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$  single crystals.

## 3. NUMERICAL SIMULATIONS

The static and dynamic behaviors of Josephson vortices in stacks of Josephson junctions were studied by numerical simulations based on Eq. (1). For the simulations we considered overlap-type junctions and used the boundary condition  $\varphi_n'(0, \tau) = \varphi_n'(l, \tau) = -b(1+2S)$ , where  $l = L/\lambda_J$  is the normalized junction length and  $b = B/(\mu_0 J_c \lambda_J)$  is the normalized magnetic field, and then took  $l = 10$  and  $S = -0.499997$ , which are similar to our experimental conditions. In order to save simulation time,  $\alpha = 0.1$  which is 3–4 times larger than our experimental value was chosen.

Figure 1 shows typical static solutions for a stack of  $N = 7$  junctions without bias current ( $\gamma = 0$ ) as a function of magnetic field  $h$  normalized by  $h_0 = \Phi_0/\lambda_J(t+d)$  at which one flux quantum is held in a single junction. In this figure, the horizontal scale is the spatial coordinate, the curves account for the distributions of Josephson currents and the circles mark the positions of Josephson vortices

stacked Josephson junction system can be described by the perturbed coupled sine-Gordon equations, which were developed by Sakai *et al.* [12]. For a stack composed of  $N$  identical junctions the equations are given by

$S = -\lambda_L/d' \sinh(t/\lambda_L)$ . Usually, for intrinsic Josephson junctions of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$ ,  $\lambda_J = 0.5\text{--}1 \mu\text{m}$  and  $\omega_0 = (3.7\text{--}5.2) \times 10^{11} \text{ rad/s}$ .

An important quantity for the characterization of the vortex dynamics in a single Josephson junction is the velocity of electromagnetic wave in it, so-called Swihart velocity  $c_{sw} = \lambda_J \omega_0$ , because this limits the velocity of vortex flow in it. According to further investigation of Eq. (1) [13,14], in the stack of  $N$  junctions  $N$  different electromagnetic modes and their corresponding velocities  $c_n$  exist. They are given by

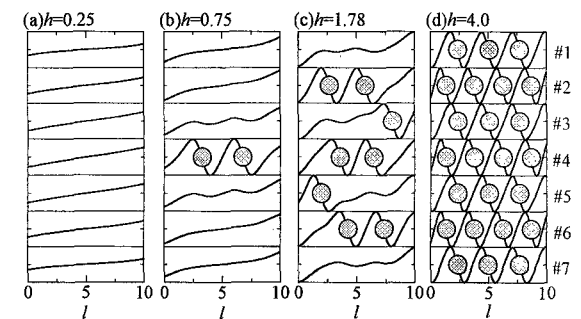


Fig. 1. Simulation of the static states of a stack of  $N = 7$  Josephson junctions ( $l = 10$ ) with  $\gamma = 0$  as a function of  $h$ .

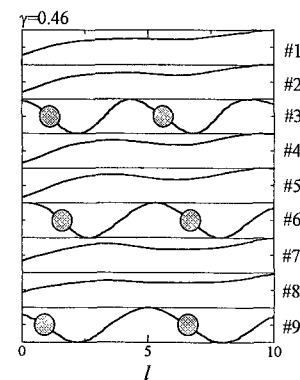


Fig. 2. A simulated snapshot of vortex flow in a stack of  $N = 9$  Josephson junctions ( $l = 10$ ) with  $\gamma = 0.46$  under  $h = 0.9$ .

in the stack. It is found that as  $h$  increases, vortices start to penetrate the middle junction of the stack at the lower critical field [15] and then the outer junctions in the stack at higher  $h$ . Furthermore, it is seen that the vortices tend to form a triangular lattice at much higher  $h$ .

On the other hand, the penetrating vortices flow in the corresponding junctions when an appropriate bias current is injected to the stack under the external magnetic field. Figure 2 shows one of the simulated snapshots of vortex flow in a stack of  $N = 9$  junctions with  $\gamma = 0.46$  under comparatively low magnetic field  $h = 0.9$ . It is observed that the vortices penetrating only 3 junctions of the stack flow preferentially with a rectangular lattice, i.e., in-phase mode.

In addition, it was also observed that when  $h$  is much higher, the vortices, penetrating all junctions of the stack as shown in Fig. 1(d), flow with a triangular lattice, i.e., out-of-phase mode, for low  $\gamma$ , but can flow with a rectangular lattice, i.e., in-phase mode, for much higher  $\gamma$ . Recently, by large scale simulations by Machida *et al.* [16], the in-phase state has been found to exist stably in a wide region in the  $I$ - $V$  characteristics for the stack of intrinsic Josephson junctions under high magnetic field.

From the simulated results, we infer that the in-phase mode of vortex flow may be stable even in stacks of intrinsic Josephson junctions under comparatively low magnetic field, under which only a few junctions of the stack become vortex-flow state.

#### 4. EXPERIMENTAL

$\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$  single crystals with critical temperatures  $T_c$  of  $\sim 85$  K were grown by a conventional melting method [17]. The grown single crystals were glued on glass substrates and then cleaved in air to obtain their fresh surfaces. Subsequently, Au thin films of 20-40 nm thicknesses were deposited on the cleaved surfaces to contact well electrically. And then, rectangular mesas with  $L$  of 10-150  $\mu\text{m}$ ,  $W$  of 2-10  $\mu\text{m}$ , and  $T$  of 0.006-0.075  $\mu\text{m}$  consisting of 4-50 intrinsic Josephson junctions were fabricated to have good uniformity on the single crystals by using standard photolithography and Ar ion milling. Their current-voltage ( $I$ - $V$ ) characteristics along the  $c$ -axis direction were measured using 3 and 4 terminal methods at 4.2 K. Magnetic fields  $B$  up to 0.2 T were applied to the mesas, parallel to the junction layers, using a solenoid coil.

#### 5. RESULTS AND DISCUSSION

Figure 3 shows the typical  $I$ - $V$  characteristic of a mesa with lateral dimensions of  $10 \times 10 \mu\text{m}^2$  with 12 junctions in the stack measured at 4.2 K without external magnetic field. In this figure, one superconducting and multiple hysteretic resistive branches which correspond to the quasiparticle branches of individual intrinsic Josephson junctions in the mesa are observed with voltage spacings of  $\sim 25$  mV. The critical currents  $I_c$  of the junctions are nearly equal and are  $\sim 90 \mu\text{A}$  which corresponds to the critical current density  $J_c$  of  $\sim 90 \text{ A/cm}^2$ . The  $I_c$  is sensitive to the applied magnetic field  $B$  and the behavior of  $I_c(B)$  is similar to that of a conventional long Josephson junction.

Figure 4 shows the  $I$ - $V$  characteristics taken repeatedly with increasing  $B$  at a step of 32.2 mT for a mesa with  $L \times W$  of  $150 \times 10 \mu\text{m}^2$  with 15 junctions in the stack. We have successfully observed Josephson vortex flow

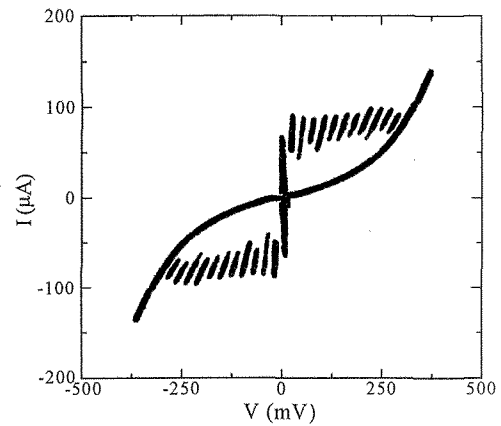


Fig. 3. Typical  $I$ - $V$  characteristic of a mesa with lateral dimensions of  $10 \times 10 \mu\text{m}^2$  with 12 junctions in the stack measured at 4.2 K without external magnetic field.

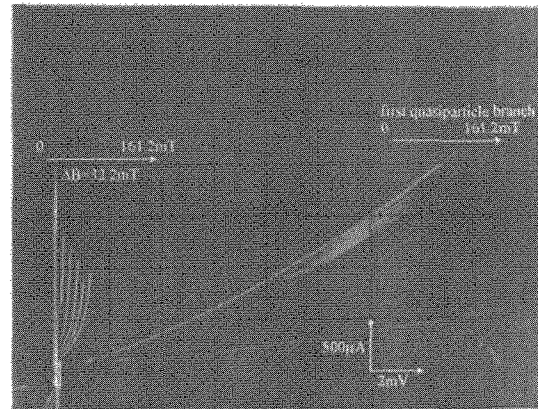


Fig. 4. Typical  $I$ - $V$  characteristics taken repeatedly with increasing  $B$  at a step of 32.2 mT for a mesa with  $L \times W$  of  $150 \times 10 \mu\text{m}^2$  with 15 junctions in the stack at 4.2 K.

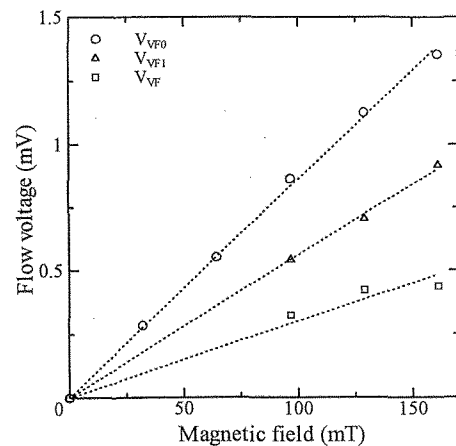


Fig. 5. Magnetic field dependence of the maximum voltages  $V_{VF0}$  and  $V_{VF1}$  of the vortex flow branches from the  $V = 0$  V (0th) branch and the 1st quasiparticle branch, respectively, observed in Fig. 4, and of the difference  $V_{VF}$  between them.

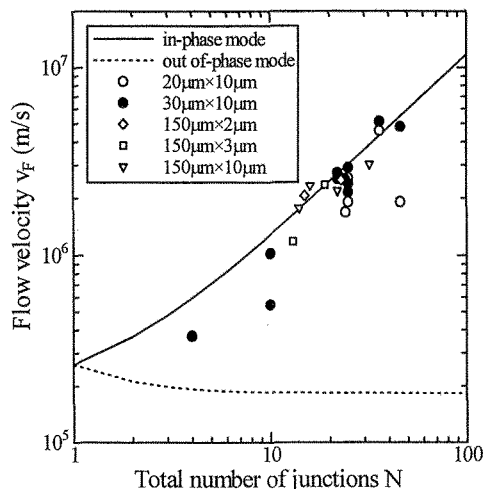


Fig. 6. Flow velocity  $v_F$  versus the total number  $N$  of intrinsic Josephson junctions in mesas.

branches appear from the original superconducting ( $V=0$  V) branch and multiple quasiparticle branches, and then shift to higher voltages in the  $I$ - $V$  characteristics, with increasing  $B$ .

Figure 5 shows the magnetic field dependence of the maximum voltages  $V_{VF0}$  and  $V_{VF1}$  of the vortex flow branches from the  $V=0$  V (0th) branch and the 1st quasiparticle branch, respectively, observed in Fig. 4, and of the difference  $V_{VF}$  between them. These voltages are observed to be proportional to  $B$ . Generally, for a stack of Josephson junctions under low  $B$  at which small number of junctions in the stack are in a vortex flow state, the vortex flow voltage per one junction can be determined from the voltage difference between vortex flow branches from  $n$ th quasiparticle branch and  $(n+1)$ th quasiparticle branch at the same  $B$  [11]. Therefore, it turns out that  $V_{VF}$  is the vortex flow voltage per one junction in the mesa shown in Fig. 4, and hence  $V_{VF0} = m V_{VF}$  as  $m$  junctions are in a vortex flow state in it. Then, the velocity of vortex flow  $v_F$  is defined as  $v_F = V_{VF}/(t+d)B$  [7]. Thus, from  $V_{VF0}/V_{VF}$  and  $dV_{VF}/dB$  we find that 3 junctions participate in vortex flow in the stack and  $v_F$  is  $\sim 1.78 \times 10^6$  m/s.

In the same manner,  $v_F$  in various mesas was examined. Figure 6 shows the relation between the observed  $v_F$  and the total number  $N$  of the junctions in the measured mesas. Here, solid and dashed lines, respectively, indicate  $c_1$  and  $c_N$  derived from Eq. (2) using typical parameters of BSCCO, corresponding to the velocities of the in-phase mode and the out-of-phase mode of vortex flow in the stacks. From this figure, it is found that  $v_F$  increases from  $\sim 3 \times 10^5$  to  $\sim 5 \times 10^6$  m/s along the solid line with increasing  $N$ , but independently of the mesa areas. In addition, the number of junctions participating in vortex flow in the mesas was estimated to be  $\sim 3$ . Therefore, in the mesas under magnetic fields less than 0.2 T, Josephson vortices flow only in a few junctions with the velocity mostly close to the highest one of the electromagnetic wave in the stack. This may arise from a synchronous motion of a low density of vortices preferentially with a rectangular lattice, i.e., in-phase mode, in the stack, as is inferred from the results of numerical simulations.

## 6. CONCLUSIONS

Josephson vortex flow at 4.2 K in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$  single-crystal mesas consisting of stacks of 4-50 intrinsic Josephson junctions under comparatively low magnetic field up to 0.2 T has been studied experimentally and numerically. It has been found that vortices flow in only  $\sim 3$  junctions in any mesa, and their flow velocity increases from  $\sim 3 \times 10^5$  to  $\sim 5 \times 10^6$  m/s with increasing the total number of junctions in the mesa. This behavior of vortices may arise from a preferential in-phase motion of a low density of vortices in different junctions in the mesa, corresponding mostly to the highest-velocity mode of electromagnetic wave in the stack, according to an inductive-coupling model based on coupled sine-Gordon equations.

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