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Temperature dependence of critical currents of small-sized intrinsic Josephson junctions in $Bi_2Sr_2CaCu_2O_y$

Michihide Kitamura, Akinobu Irie and Gin-ichiro Oya
Department of Electrical and Electronic Engineering,
Utsunomiya University, 7-1-2 Yoto, Utsunomiya, Tochigi 321-8585, Japan
Fax: 81-28-689-6102, e-mail: kitamura@cc.utsunomiya-u.ac.jp

The temperature dependence of critical currents $I_c(T)$ of small-sized intrinsic Josephsonjunctions in $Bi_2Sr_2CaCu_2O_y$ with a d-wave symmetry superconducting gap (SG) is calculated, and compared with the behavior of $I_c(T)$ experimentally obtained, which deviates from the Ambegaoker-Baratoff relation. The first calculation, which does not include the effect of thermal fluctuation, was not enough to explain the experiment, especially for the small current case. Then, by adding Lee's model, which includes the effect of thermal noise, into our model, we calculate the normalized current $I_c(T)/I_c(0)$ and see that Lee's model is valid for *only* the explanation of the experimental trend. We introduce here an empirical rule and show that this rule is valid for the explanation of the overall profile of all the normalized experimental curves.
Key words: Intrinsic Josephson junction, Critical current, Temperature dependence

It is well known that strongly anisotropic high-T_c superconductors (HTSC's) such as $Bi_2Sr_2CaCu_2O_y$(BSCCO) and $Tl_2Ba_2Ca_2Cu_3O_y$ (TBCCO) show an intrinsic Josephson effect, so that these single crystals can be regarded as a stack of atomic-scale Josephson junctions, so-called "intrinsic Josephson junctions (IJJ's)"[1-6]. It is now established that c-axis transport of IJJ's is due to tunneling process [5-10]. However, previous experiments using mesas with the lateral dimensions of a few tens μm^2 have several problems such as an internal heating due to the injection of quasiparticle into an extremely thin electrode [11] and non-uniformity of bais current arising from the short Josephson penetration depth of $\sim 1\ \mu m$ [1-2] compared with that of a conventional Josephson junction. In order to avoid these problems, Irie and Oya (IO) made small-sized IJJ's in BSCCO, and carefully carried out the experiments of pair and quasiparticle tunneling characteristics along the c-axis, to see the inherent characteristics of IJJ's [12]. IO found that (1) well defined gap structure and the normal resistance region can be successfully observed at 4.2 K, (2) IJJ's in BSCCO have a d-wave feature and (3) pair transport along the c-axis, i.e., Josephson current, is due to coherent interlayer tunneling. In the present paper, we theoretically consider the temperature dependence of critical currents, $I_c(T)$, of the small-sized IJJ's in BSCCO.

Very recently, we have theoretically studied the angular dependence of c-axis critical currents of BSCCO cross junctions [13], in which a general expression to calculate the temperature T and the cross angle α dependences of the c-axis pair tunneling critical current $I_c(T, \alpha)$ of a BSCCO junction crossed with an angle α has been presented, and the calculations of the α dependences of coherent and incoherent tunneling currents have been done at T=5 K. By comparing our calculated results with the experiment done by Takano et al. [14], we have concluded that (1) the c-axis pair tunneling occurs coherently, (2) a mixed superconducting gap (SG) that the d and s wave symmetry SG's are mixed together must be used on the study of the symmetry sensitive superconducting property such as the α dependence of the $I_c(T, \alpha)$, and (3) a calculated result which well reproduces the experimental fact is obtained when both the mixed SG and the effect of Andreev reflection are considered. Our conclusion (1) is the same as the conclusion (3) by IO, so in the following we consider only the coherent tunneling, i.e., an in-plane momentum is conserved on the CP tunneling from left superconductor to right one.

In the present paper, we consider the temperature T dependence of the c-axis pair tunneling critical current $I_c(T)$ of BSCCO with $\alpha = 0$. It may be reasonable to suppose that the $I_c(T)$ does not so strongly depend on the mixing rate ε of s-wave symmetry SG, since there is an experimental fact that the temperature variation of the normalized SG, $\Delta(T)/\Delta(0)$, shows the BCS-like behaviour irrespective of the symmetry of SG [12,15,16]. In the following, therefore, we consider the case in which there is no s-wave symmetry SG, i.e., $\varepsilon = 0$.

On equation (30) in our recent paper [13], by setting both the α and ε to 0, the coherent tunneling current

$I_c(T,0,0,\eta)_{coherent} \equiv I_c(T,\eta)_{coherent}$ is written as

$$I_c(T,\eta)_{coherent} = \left| B \sum_k^{1stBZ} Ang\{\Delta_k^* \Delta_k\} \sum_{k'}^{1stBZ} \Lambda_c(k,k',T) A_R(\hat{k},\hat{k}',\eta) \right| \quad (1)$$

where B is a constant, $Ang\{\Delta_k^* \Delta_k\}$ is equal to $|\chi(\hat{k})|^2$ because of $\varepsilon=0$, and $\Lambda_c(k,k',T)$ is given as

$$\Lambda_c(k,k',T) = \frac{\Delta(T)^2 \chi(\hat{k})\chi(\hat{k}')}{E_k E_{k'}} \times \left[\frac{f(E_k)-f(E_{k'})}{E_k - E_{k'}} + \frac{1-f(E_k)-f(E_{k'})}{E_k + E_{k'}} \right] \quad (2)$$

where $\chi(\hat{k})$ is the angular part of the d-wave symmetry SG Δ_k, and E_k is the quasiparticle excitation energy. The $A_R(\hat{k},\hat{k}',\eta)$ in Eq.(1) is a term describing the current change due to the Andreev reflection. In general, the $A_R(\hat{k},\hat{k}',\eta)$ may depend on the temperature T, however, in the present paper we assume for simplicity that this term makes no sizable effect on the temperature dependence of the critical current. Therefore, the term $A_R(\hat{k},\hat{k}',\eta)$ is set to 1 on the present consideration, so that in the following we regard the $I_c(T,\eta)_{coherent}$ with $A_R(\hat{k},\hat{k}',\eta)=1$ as the temperature dependent c-axis critical current $I_c(T)$, that is,

$$I_c(T) = \left| B \sum_k^{1stBZ} |\chi(\hat{k})|^2 \sum_{k'}^{1stBZ} \Lambda_c(k,k',T) \right| \quad (3)$$

where a wavenumber vector **k'** in right superconductor is represented as (k_x,k_y,k_z') when the wavenumber vector **k** in left superconductor is written as (k_x,k_y,k_z) because of coherent tunneling.

Normalized critical current $I_c(T)/I_c(0)$ ($\equiv I_c^{(Nor)}(T)_{cal}$) calculated from Eq.(3) is shown in Fig.1 together with the experimental ones $I_c^{(Nor)}(T)_{exp}$ for junction cross sections S with 1, 4 and 25 μm^2 observed by IO [12]. For the reference, the normalized critical current $I_c^{(Nor)}(T)_{AB}$ calculated from Ambegaoker-Baratoff (AB) relation [17] using the $\Delta(T)$ of BSCCO is also shown. It is clear that the critical current at 4.2 K, $I_c(4.2)(\cong I_c(0))$, decreases with decreasing S. IO observed that $I_c(4.2)$ is 7.8~8.4 μA for S=1 μm^2, 30~33 μA for S=4 μm^2, and 240~250 μA for S=25 μm^2. Figure 1 clearly shows that not only the $I_c^{(Nor)}(T)_{cal}$ but also the $I_c^{(Nor)}(T)_{AB}$ can not explain the experimental facts that the overall profile of $I_c^{(Nor)}(T)_{exp}$ largely changes due to the change of S, that is, change of total current $I_c(T)$, and that an explicit exponential tail appears at higher temperature region as S decreases. Namely, it is observed that the disagreement between the experiment and theory spreads out due to the decreasing of $I_c(T)$. This is not surprising because

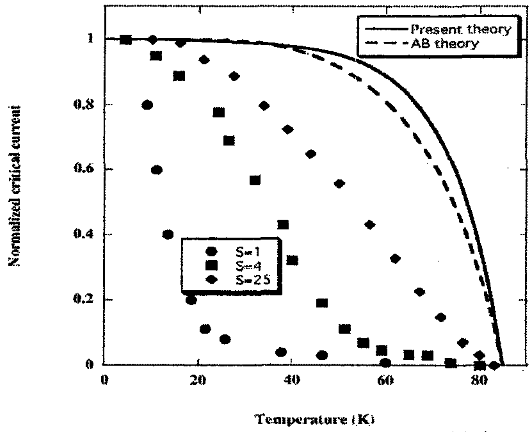

Fig.1 Normalized critical current $I_c^{(Nor)}(T)_{cal}$ calculated from Eq.(3), normalized experimental ones $I_c^{(Nor)}(T)_{exp}$ for S with 1, 4 and 25 μm^2, and the normalized critical current $I_c^{(Nor)}(T)_{AB}$ calculated from Ambegaoker-Baratoff (AB) relation using the $\Delta(T)$ of BSCCO.

both the present and AB theories do not include the effects of thermal fluctuation such as a thermal noise and a competition process of Josephson coupling energy $E_J(T)$ given by $(\hbar/2e)I_c(T)$ and thermal energy k_BT.

Lee studied the effect of thermal noise on the current-voltage characteristics of a Josephson junction on the basis of resistivity shunted junction (RSJ) model [18]. In the study, he presented an analytical expression of the mean voltage <V> which is a voltage observed experimentally on the presence of the thermal noise at a finite temperature T. For the $RC(2eI_c(T)/\hbar c)^{1/2}(\equiv \Omega(T))$ calculated from the resistance R, the capacitance C and the critical current $I_c(T)$, he presented two expressions, one is for $\Omega^2 \ll 1$ and the other is for $\Omega^2 \gg 1$. For IJJ's of S=1, 4 and 25 μm^2 in BSCCO samples IO made, the averaged normal resistances R_N per one junction were 245, 65 and 18.5 Ω, respectively, and the capacitances C calculated as $\varepsilon\varepsilon_0 S/d$ were 0.059, 0.236 and 1.476 pF, respectively, using ε=8, ε_0=8.854×10^{-12} F/m, and d=12 Å. The curves of $\Omega(T)^2$ calculated by using R_N, C and $I_c(T)=I_c(0)I_c^{(Nor)}(T)_{cal}$ of BSCCO are shown in Fig.2 for junctions with S=1, 4 and 25 μm^2. Except for the vicinity of the critical temperature T_c(=85K), figure 2 explicitly shows that the condition of $\Omega(T)^2 \gg 1$ is satisfied on all the temperature region in curve for S=25 μm^2, and for S=1 and 4 μm^2 curves its condition is fairly well satisfied on the temperature region less than about 70 K. According to Lee, the mean voltage <V> in the case of $\Omega^2 \gg 1$ is given by [18]

$$\frac{1}{(2e/h)<V>RC} = \int_0^{\gamma(T)|E_m(\alpha,\gamma)|/2} \frac{e^t - 1}{t} dt \quad (4)$$

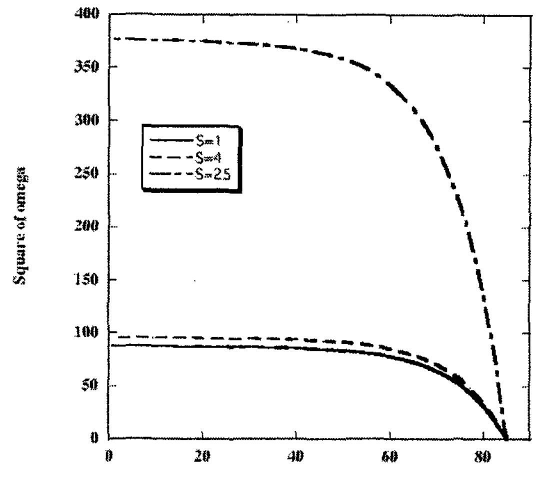

Fig.2 Curves of $\Omega(T)^2$ calculated by using R_N, C and $I_c(T)=I_c(0)I_c^{(Nor)}(T)_{cal}$ of BSCCO. Those are for junctions with S=1, 4 and 25 μ m^2.

where $\gamma(T)/2=\hbar I_c(T)/2ek_BT=E_J(T)/k_BT$ and $E_m(\alpha_r)$ is given by

$$E_m(\alpha_r)=\left[\alpha_r\left(\pi-2\sin^{-1}\alpha_r\right)-2\left(1-\alpha_r^{\,2}\right)^{1/2}\right] \qquad (5)$$

Here α_r is defined by

$$\alpha_r\equiv\frac{I_c(T)_{\exp}}{I_c(T)_{cal}}=\frac{I_c^{(Nor)}(T)_{\exp}}{I_c^{(Nor)}(T)_{cal}}=\alpha_r(T) \qquad (6)$$

so it is noted that $\alpha_r(T)>0$, $\alpha_r(0)=1$ and the $\alpha_r(T)$ decreases with increasing T. We wish to know the critical current $I_c(T)_{exp}$, i.e., $\alpha_r(T)$, on the presence of the thermal noise at a finite temperature T. The critical current state can be regarded as an infinitesimal voltage state. In Lee's model, it is impossible to set the value of $\langle V\rangle$ to 0, so that we introduce here a parameter $\langle\zeta\rangle$ defined by $\langle V\rangle/I_c(T)_{cal}R_N$. The value of $\langle\zeta\rangle$ is set to 10^{-3} tentatively, so that the $\langle V\rangle=\langle\zeta\rangle I_c(T)_{cal}R_N$ is less than $10^{-3}\times I_c(0)R_N$ which is 2~5$\times10^{-6}$ V for BSCCO. The $I_c^{(Nor)}(T)_{Lee}$ calculated on the basis of Lee's model are shown in Fig.3 for junctions with S=1, 4 and 25 μ m^2, together with the experimental data $I_c^{(Nor)}(T)_{exp}$ of S=1, 4 and 25 μ m^2 junctions, and the $I_c^{(Nor)}(T)_{cal}$ by present model for the reference. Figure 3 shows that the effect of thermal noise, i.e., the deviation of $I_c^{(Nor)}(T)_{Lee}$ from $I_c^{(Nor)}(T)_{cal}$, is observed on all cases, and its effect increases with decreasing the junction cross section S, i.e., decreasing the Josephson coupling energy $E_J(T)$. This trend is also observed on the experimental data $I_c^{(Nor)}(T)_{exp}$ shown in Fig.3, however, the $I_c^{(Nor)}(T)_{Lee}$ are far from the $I_c^{(Nor)}(T)_{exp}$. Namely, we can say that Lee's model which includes the

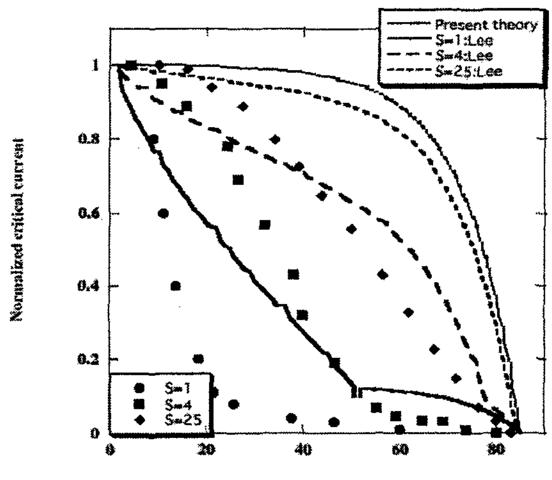

Fig.3 Curves of normalized critical currents $I_c^{(Nor)}(T)_{Lee}$ calculated by adding Lee's model into our expression. Those are for junctions with S=1, 4 and 25 μ m^2. Normalized experimental data $I_c^{(Nor)}(T)_{exp}$ for S=1, 4 and 25 μ m^2 junctions and the normalized critical current $I_c^{(Nor)}(T)_{cal}$ calculated without the inclusion of Lee's model, i.e., that calculated from Eq.(3) are also shown.

effect of thermal noise is valid for *only* the explanation of the experimental trend. If the term $A_R(\hat{k},\hat{k}',\eta)$ describing the effect of the Andreev reflection shows the temperature dependence, and if we know it, present calculations may be improved. However, we have now no idea for it, in the following, therefore, we introduce an empirical rule for $\alpha_r(T)$ to expalin the experimental data.

Equation (6) tells us that $I_c(T)_{\exp}=\alpha_r(T)I_c(T)_{cal}=\alpha_r(T)I_c(0)I_c^{(Nor)}(T)_{cal}$ and as already stated, the $\alpha_r(T)$ is a function such that $\alpha_r(T)>0$, $\alpha_r(0)=1$ and the $\alpha_r(T)$ decreases with increasing T. Therefore, we assume that the function $\alpha_r(T)$ is given by $\exp\{-(T/T^*)^\ell\}$, so that the $I_c(T)_{exp}$ is represented as

$$I_c(T)_{\exp}=\exp\{-(T/T^*)^\ell\}I_c(0)I_c^{(Nor)}(T)_{cal}\equiv I_c(T)_{emp} \qquad (7)$$

where $\ell\geq1$. T^* is a temperature to characterize the temperature variation of $I_c(T)_{emp}$. There is a temperature T_J at which the experimental Josephson coupling energy is equal to the thermal energy [12], that is, $E_J^{(\exp)}(T_J)=(\hbar/2e)I_c(T_J)_{\exp}=k_BT_J$, so that the T^* can be decided from

$$\exp\{-(T_J/T^*)^\ell\}=\frac{k_BT_J}{E_J(0)I_c^{(Nor)}(T_J)_{cal}} \qquad (8)$$

where $E_J(0)=(\hbar/2e)I_c(0)$. IO observed that the T_J in K is about 20 and 55 for IJJ's in BSCCO with S=1 and 4 μ m^2 and extrapolated that the T_J is

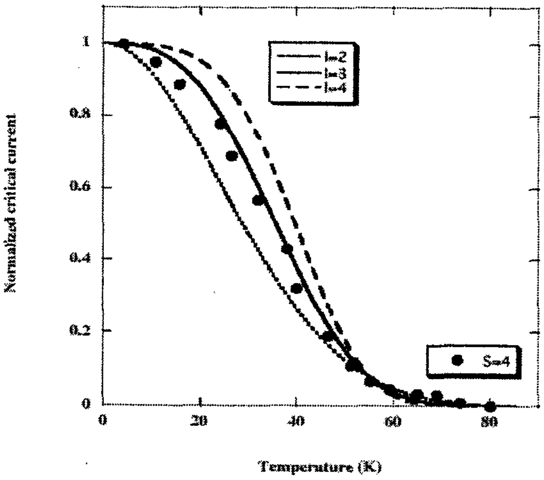

Fig.4 Curves of normalized critical currents $I_c^{(Nor)}(T)_{emp}$ for the junction with S=4 μ m^2 calculated from the empirical model Eq.(7). Those are for ℓ=2, 3 and 4 calculated by using 55 K as T_J. Normalized experimental data for S=4 μ m^2 junction are also shown.

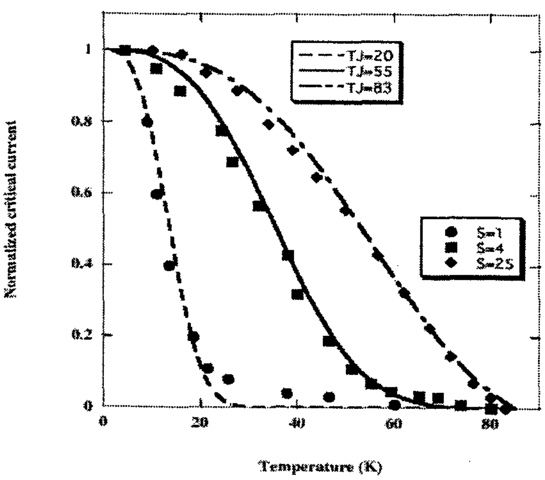

Fig.5 Curves of $I_c^{(Nor)}(T)_{emp}$ calculated by setting ℓ to 3 and using 20, 55 and 83 K as the values of T_J. The experimental data are also shown.

about 80 K for S=25 μ m^2 IJJ [12]. First, in order to see how the $I_c^{(Nor)}(T)_{emp}\left(\equiv I_c(T)_{emp}/I_c(0)\right)$ calculated from the present empirical model Eq.(7) is changed due to the change of ℓ, we have calculated three $I_c^{(Nor)}(T)_{emp}$ for the junction with S=4 μ m^2 which are for ℓ=2, 3 and 4. Results calculated by using 55 K as T_J are shown in Fig.4 together with the experimental data for S=4 μ m^2 junction. It is seen that the result calculated by using ℓ=3 fairly well reproduces the experiment data. It may be possible to adjust the value of ℓ so as to fit the calculated result to the experimental ones, however, we don't do it. Therefore, in the following, the value of ℓ is set to 3.

Three $I_c^{(Nor)}(T)_{emp}$ calculated by using 20, 55 and 83 K as T_J, thus the corresponding T^* are 15.6, 40.8 and 62.7 K, respectively, are shown in Fig.5 together with the experimental data of S=1, 4 and 25 μ m^2 junctions. From this figure, we can observe that our expression added an empirical rule based on only an adjustable parameter ℓ fairly well reproduces all the experimental data presented here. Here we wish to comment that the $I_c^{(Nor)}(T)_{emp}$ calculated with (ℓ,T_J)=(3,83) gives a better result rather than that calculated with (ℓ,T_J)=(3,80). This may be caused by the fact that IO have decided the value of T_J of S=25 μ m^2 junction from not directly but an extrapolation.

In the present paper, we presented an empirical rule to explain the temperature T dependence of critical current $I_c(T)$ of BSCCO. This is just an empirical, but we think that this rule gives a guide line on the further study of the T-dependence of the $I_c(T)$.

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