Transactions of the Materials Research Society of Japan 29 [8] 3791-3794 (2004)

Temperature-Dependent Mechanical Behaviors of High Damping Rubber Material and its Modeling

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Abstract: Temperature dependency behavior of High Damping Rubber material (HDR) and its thermo-mechanical constitutive modeling are studied. The use of infrared thermographs to measure temperature field in HDR under cyclic shear is presented. Changes of mechanical properties of HDR are investigated with respect to frequency of loading and rubber body temperature. Based on the results of rubber material tests, a temperature dependent constitutive model for HDR is proposed, which combines elasto-plastic body with strain dependent isotropic hardening law and hyperelastic body with damage model. Temperature dependent parameters, which can express rate dependent hardening, yielding and thermal softening are introduced to the model. The proposed constitutive model can express both temperature dependent and frequency dependent behavior of HDR. Finally, the energy balance equation is introduced to evaluate average surface temperature of the material. The modeling results agree well with the experimental ones.

Key words: *high damping rubber, thermographs, temperature field, constitutive model, thermal balance*

1. INTRODUCTION

Rubber has many useful applications in engineering because of its special properties. Nowadays, many important products that meet high requirements are made of high damping rubber (HDR), such as vibration mounts, bearings for bridges and for seismic isolations. HDR has high damping property compared to the natural rubber (NR). The HDR in bearings can be subjected to high frequency dynamic excitations, especially during earthquakes and dissipated considerable amount of heat increasing its body temperature. Besides, rubber in bearings subjects to various seasonal changes of temperatures. Further, strain rate effect presents in real behavior of rubber due to its viscous properties. Both thermal effect and rate effect are directly proportional; high rate of loading produces higher body temperature during cyclic excitation. Therefore, rate effects can be considered in terms of temperature. As a result, mechanical properties of those rubber materials strongly depend on its body temperature [1-3]. Hence, coupling of mechanical and thermal effects should be taken into account in modeling and in rational design procedures, especially HDR products that are subjected to dynamic excitations. An example, Fig. 1 shows the change of hysteresis loops of a HDR bearing during the same frequency of loadings but different temperatures.

So far, few researches have been conducted to investigate the thermal effects on mechanical modeling of rubber [1]. Besides, a few experimental investigations are available for studies, especially because measurement of temperature is tedious work under large deformation due to generation of large strains (more than 10%), where thermo-couples are not available. Fukahori et al. [2], Yamashita et al. [3] and Bilgili et al. [4] have studied thermal effects on large deformation behavior of carbon filled NR, although they did not consider the effects of energy absorbing properties.

The aim of this study is to experimentally investigate thermo-mechanical behavior of HDR and finally produces a thermo-mechanical constitutive model. The use of infrared thermographs to measure temperature field in HDR under cyclic shear is presented. Changes of mechanical properties of HDR are investigated with respect to frequency of loading and rubber body temperature. Then, a thermo-mechanical constitutive model for HDR is proposed, which is the extension of the model proposed by Yoshida et al. [5]. Finally, the proposed model is evaluated by an energy balance equation with average surface temperature.

2. EXPERIMENT

Experiments in displacement control cyclic simple shear were performed at different frequency with different amplitudes of loadings. Details of the performed experiments are given in Table I, and experimental set up is illustrated in Fig.1. Outputs are load-displacement relation and surface temperature field of the deformed specimens. Outside temperature of the material specimen was measured using thermo-couple, which showed that temperature was almost constant during the loading.

Table I Experimental Conditions

Type of rubber material	HDR
Shear Modulus [MPa]	0.98
Input displacement	Cyclic triangle
Loading frequency [Hz]	1, 0.1, 0.01, 0.001
Amplitude (shear strain [%])	50, 141, 264, 346
Number of cycles	3

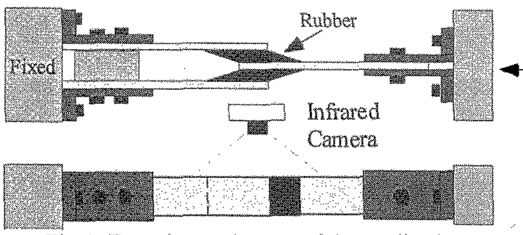

Fig.1 Experimental setup of the cyclic shear test

2.1 Measurement of temperature field

In our study, the device called infrared camera (Nikon thermal vision LAIRD3A-S) is employed for the temperature field measurements. Infrared thermographs can be obtained from the device in real time so that temperature can be measured at any point of the surface of the body. Our thermal measurement system is capable to capture 60 thermal images per second. Fig.2 shows a thermal image, which was captured during the cyclic shear deformation of HDR specimen.

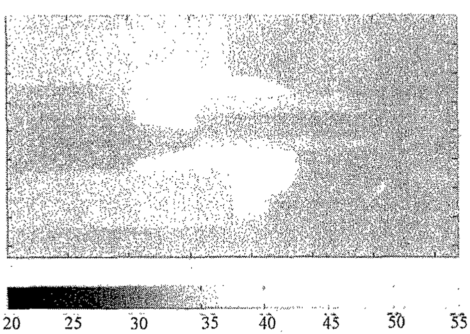

Fig.2 Temperature field under cyclic shear deformation

2.2 Experimental results

Fig.3 shows stress-strain relation for cyclic shear loading for two different frequencies. It is found that hardening and initial yielding of the hysteresis loops at different frequency with different amplitudes of loadings are changing. Fig.4 shows an example of total energy dissipation (area of hysteresis loop) versus loading maximum shear strains, for different frequency of loadings. The results describe that these variation are directly related to changing the properties of rubber such as stiffness and damping.

Fig.5 shows variation of maximum temperature of rubber surface with time at different frequency of loading with different amplitudes. It is clear that as time goes, heat losses to the outside. The increase of the temperature of the rubber body is 7° C at frequency of 0.1 Hz, and 3° C at 0.001 Hz, while, at 1 Hz, temperature increases about 20° C from the initial due to cumulative heat dissipation within short period, which causes the afore-mentioned changes of the mechanical properties of the material.

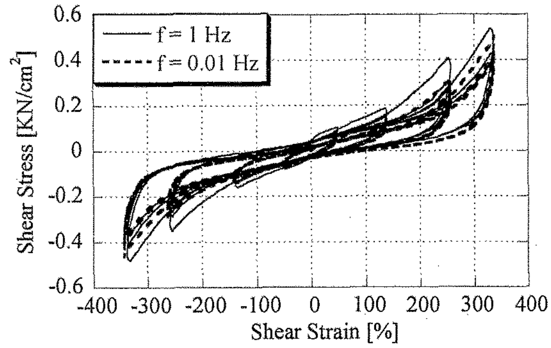

Fig.3 Stress-strain relations

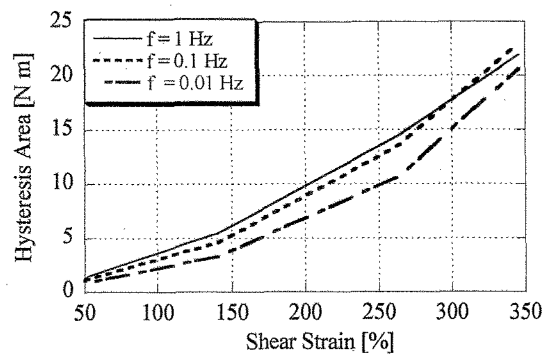

Fig.4 Energy dissipation (Second cycle of loading)

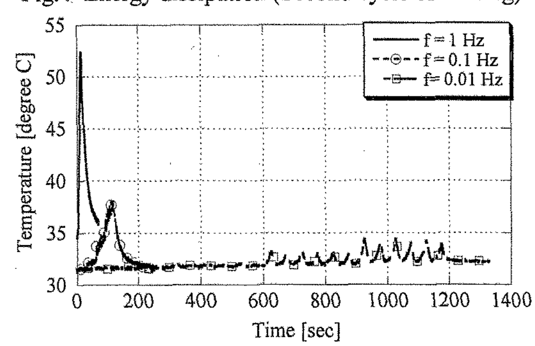

Fig.5 Maximum temperature at the surface

3. CONSTITUTIVE MODELING

A macroscopic constitutive model of HDR was first proposed by Yoshida et al. [5]. The model has combined hyperelasticity and elasto-plasticity in parallel, where viscosity is replaced by the equivalent plasticity. In this study, this model is extended to represent the thermal behavior by introducing the temperature dependent parameters into the model.

3.1 Hyperelastic Part

In the original model, they expressed total stain energy with two strain energy density functions; W_1 and W_2, which represent the linear and hardening behavior respectively. Further, each strain energy function introduces damage function (g and h) to represent the maximum stain dependency. The complete formulations for the model are given as follows.

$$\bar{W} = gW_1 + hW_2 \quad (1)$$

$$W_1 = c_1(\bar{I}_c - 3) + c_2(\bar{II}_c - 3) \quad (2)$$

$$W_2 = \frac{c_3 c}{n+1}\left(\frac{\bar{I}_c - 3}{c}\right)^{n+1} \quad (3)$$

$$g(x) = \beta + (1-\beta)\left[\frac{1 - e^{-x/\alpha}}{x/\alpha}\right] \quad (4)$$

$$h(y) = 1 - \frac{1}{1 + \exp\{-a_H(y - b_H)\}} \quad (5)$$

$$y(t) = \max_{s\in(-\infty,t]}\{\bar{I}_c(s) - 3\},\ x(t) = \max_{s\in(-\infty,t]}\sqrt{2W_1(s)} \quad (6)$$

where $\bar{I}_c$ and $\bar{II}_c$ are reduced first and second invariant of right Cauchy green tensor respectively; c_1, c_2, c_3, c, n, α, β, a_H and b_H are material constants.

In this study, to represents thermal effects, the model is improved with an addition strain energy density function W_3 as follows.

$$\bar{W} = gW_1 + hW_2 + W_3 \quad (7)$$

$$W_3 = a_\theta \frac{\theta}{\theta_0}(\bar{I}_c - 3)^{b_\theta} - c_\theta\theta(\bar{I}_c - 3)^{d_\theta} \quad (8)$$

where $a_\theta, b_\theta, c_\theta, d_\theta$ are material constants, and θ, θ_0 are temperature of material and room temperature respectively. In W_3, the first term controls the rate dependent hardening, while the second controls the thermal softening [6].

3.2 Elasto-plastic with thermal effect

The elasto-plastic part in the original model is described by a differential equation as follows.

$$\dot{\mathbf{T}}_{(J)} = \mathbf{C}^{(E)} : (\mathbf{D} - \mathbf{D}^P) \quad (9)$$

$$\mathbf{D}^P = (3k_2)^{1/2}(3J_2)^{(N-1)/2}\frac{\mathbf{T}'}{\tau_y} \quad (10)$$

$$k_2 = \frac{\mathbf{D}' : \mathbf{D}'}{2},\ J_2 = \frac{\mathbf{T}' : \mathbf{T}'}{2\tau_y^2} \quad (11)$$

where N are material constant, and τ_y is the current yield stress, which has been expressed as a functions of I_c in order to describe the hardening behavior.

In this study, the yield stress is considered as not only a function of strain invariants, but also a function of temperature as described in the following equations.

$$\tau_y = \tau'(\theta, \theta_0)\left\{1 + \left(\frac{\bar{I}_c - 3}{c}\right)^b\right\},\ \tau' = (\tau_0 / e_\theta)\frac{\theta}{\theta_0} \quad (12)$$

where τ_0, b, e_θ are initial yield stress and two material constants respectively. In addition, the elastic constitutive tensor ($C^{(E)}$), which is described with hyperelastidity, is also updated by including the temperature effect. Complete derivation of elastic tensor with W_E is shown as follows.

$$C^{(E)}_{pqrs} = \frac{1}{J}F_{pi}F_{qj}F_{rk}F_{sl}C^{(0)}_{ijkl} + \delta_{pr}T^{(h)}_{sp} + \delta_{qs}T^{(h)}_{pr} - \delta_{rs}T^{(h)}_{pq} \quad (13)$$

$$C^{(0)} = \frac{\partial^2 W_E}{\partial\mathbf{E}\partial\mathbf{E}}, \quad (14)$$

$$\mathbf{T}^{(h)} = \frac{1}{J}\mathbf{F}.\frac{\partial W_E}{\partial\mathbf{E}}.\mathbf{F}^T \quad (15)$$

$$W_E = c_4(\bar{I}_C - 3) + c_5(\bar{II}_C - 3) + \frac{c_4 c}{m+1}\left(\frac{\bar{I}_C - 3}{c}\right)^{m+1} + \frac{f_\theta\theta}{\theta_0}(\bar{I}_C - 3) \quad (16)$$

where c_4, c_5, m and f_θ are material constants. Fig.6a and 6b show the comparison of the experimental results with a model. They show the good agreement except virgin loading. Virgin loading is not modeled because usually all rubber bearings are subjected to virgin loading before installations.

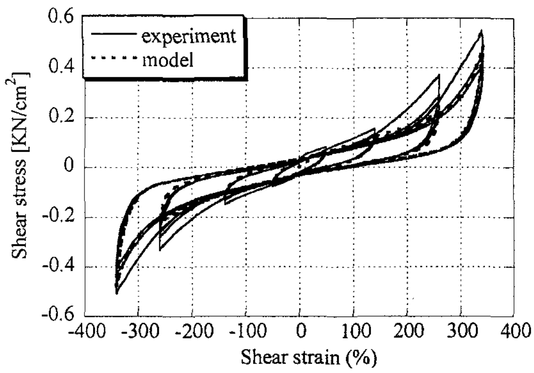

Fig.6a Stress-strain (f=0.1 Hz)

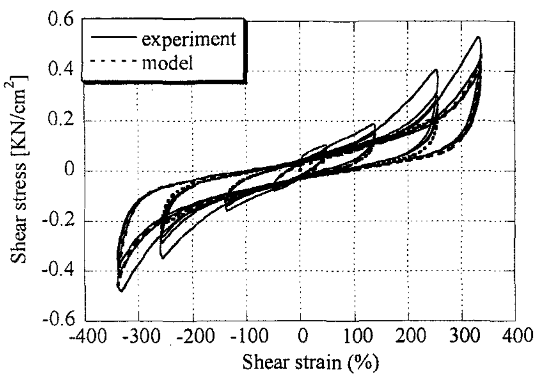

Fig.6b Stress-strain (f=1 Hz)

3.3 Temperature evaluation

In order to evaluate the average temperature using the model, thermal balance equations is introduced. The equation of conservations of energy in Eulerian variables in current configuration is described as follows [7].

$$\frac{d}{dt}\int_{D(t)}\rho(\mathbf{X},t)e(\mathbf{X},t)dV = \int_{D(t)}[r(\mathbf{X},t) + \mathbf{T}(\mathbf{X},t) : \bar{\mathbf{D}}(\mathbf{X},t)]dV - \int_{\partial D(t)} < \mathbf{q}(\mathbf{X},t) > \{\mathbf{N}(\mathbf{X},t)\}dS \quad (17)$$

where $\mathbf{X}$ is positions vector, ρ is a density, e represents internal energy, r is production rate of energy from outside, $\mathbf{T}$ is Cauchy stress tensor, $\bar{\mathbf{D}}$ is deformation rate tensor, **q** is the heat flux by conduction and **N** is a normal vector. In this problem, we assumed convective heat loss at the boundaries and no local conduction. Possible local conduction is also given by equivalent convection. There is no outside energy production. Internal production of energy results from irreversible work is simply calculated by time integration of stress-strain produce in the model and subsequently estimated average temperature is used to calculate the stress in the next step. It is assumed that 95% of irreversible work is converted into the heat. The simplified energy balance equation is given as follows

$$\rho C\frac{d\theta}{dt} = Q_{gen} - \frac{hA}{V}(\theta - \theta_0) \quad \textbf{(18)}$$

where C is specific heat capacity of material, h is convective heat transfer coefficient, V is volume, A is convective area, and average heat generation is

$$Q_{gen} = \frac{0.95}{T}\int_0^T \tau d\gamma \quad \textbf{(19)}$$

where τ is shear stress, γ is shear strain and T is the period of one cycle loading.

The solution of differential equation in Equation (19) produces average surface temperature. In the analysis, $\rho C = 1.2\times 10^6 \text{jm}^{-3}\text{K}^{-1}$ and $h = 5\ \text{Wm}^{-2}\text{K}^{-1}$ are employed. Fig.7a shows rate of heat production given by the model and experiments while Fig.7b shows temperature evaluation by model and comparison with experimental results. Both graphs show that temperature evaluation model can be used to predict average surface temperature of the HDR.

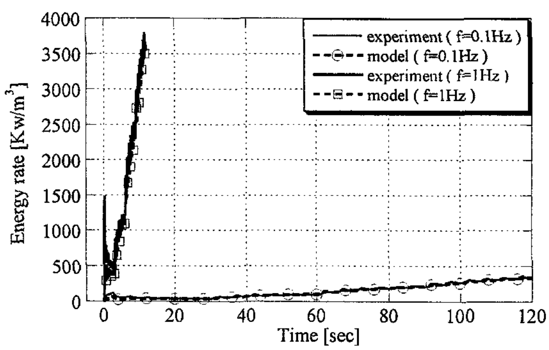

Fig.7a Rate of heat production

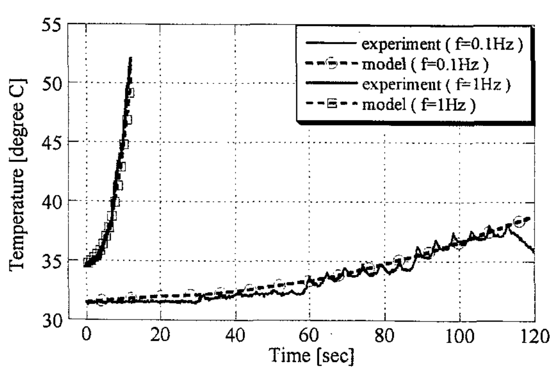

Fig.7b Temperature comparison

4. CONCLUSIONS

In this article, we have presented experimental investigations and modeling of thermo-mechanical behaviors of HDR material. Following conclusions can be drawn from the article.

(1) The infrared thermographs can be successfully used to measure temperature field of largely deformed material.

(2) The experimental result of cyclic behavior of HDR has shown that material has both rate dependency and temperature dependency.

(3) A constitutive model is proposed for HDR. The model consists of energy absorbing properties, hardening properties, viscosity properties and thermal effects. The model shows good agreement with experimental results except virgin loading.

(4) The proposed thermal coupling equation can be used to evaluate the average surface temperature of the material parallel with stress evaluation. Average temperature calculated by the model shows reasonable agreement with experimental results.

This study would be used as a key for thermo-mechanical finite element modeling for high damping rubber bearings.

ACKNOWLEDGEMENTS

The Authors wish to acknowledge Mr. Morishige and other staff members of Nippon Chuzo Company, Japan for providing testing materials. This research is co-supported by the Ministry of Education, Culture, Sports, Science and Technology, Japan.

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(Received October 13, 2003; Accepted April 24, 2004)