# Polymer Networks including Ring Forming Parameter in the Cure of Diglysidyle Ether of Bisphenol A and Propylene Glycol Diamino Ether 

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#### Abstract

A molecular model was constructed for the network formation through the polymerisation of $\mathrm{RA}_{4}{ }^{+}$ $\mathrm{RB}_{2}$ type reaction. The model was intended to be applied for the gel point prediction in the curing of diglysidyle ether of bisphenol A and propylene glycol diamino ether which correspond with $\mathrm{RA}_{4}$ and $\mathrm{RB}_{2}$ respectively. The ring forming parameter was incorporated into the model to improve the classical gelation theory; the parameter is the ratio of internal and external reactant groups, and was used to measure the competition between inter- and intramolecular reaction. Specific reactions of $\mathrm{RA}_{4}$ and $\mathrm{RB}_{2}$ were considered to give the internal concentration of reactant groups.


Key words: ring structure, network formation, molecular model, gel point, epoxy, cure

## 1. INTRODUCTION

An experimental research work has so far been carried out in our Laboratory on the polymerisation of diglysidyle ether of bisphenol A (DGEBA) and propylene glycol diamino ether (PPG diamine) in terms of network formation and gel point estimation.[1] Polymerisation of diamine and diepoxy molecules has been extensively researched by many research groups.[2-5]

In order to make better interpretation on our experimental results, it was shown to be essential to estimate the quantity of the ring structures which do not contribute to the elasticity but exist in the network formed by the polymerisation.[6] The quantity of the ring structures mean the lengths and the numbers of the polymer chains forming them. Although this estimation is considered to be very difficult, to build up a suitable model will help to carry it out.[7] A summary of the previous study was shown below.
(1) According to the proceeding of the reaction, H atoms of the amine group were categorised as $\mathrm{H} 1, \mathrm{H} 2, \mathrm{HR}$, where the atoms which belong to the unreacted amine groups were defined as H 1 . The unreacted hydrogen atoms which belong to the semireacted amine groups were defined as H2. The reacted hydrogen atoms were defined as HR. The progress of the reaction was expressed by the fraction of these H atoms, $p_{\mathrm{H} 1}, p_{\mathrm{H} 2}$ and $p_{\mathrm{HR}}$, where $p_{\mathrm{H} 1}=\left(C_{\mathrm{H} 1} / C_{\mathrm{H} 1}{ }^{0}\right)$, $p_{\mathrm{H} 2}=\left(C_{\mathrm{H} 2} / C_{\mathrm{H}_{0}}{ }^{0}\right)$ and $p_{\mathrm{HR}}=\left(C_{\mathrm{HR}} / C_{\mathrm{H} 1}{ }^{0}\right) ; C_{\mathrm{H} 1}, C_{\mathrm{H} 2}$, $C_{\mathrm{HR}}$ and $C_{\mathrm{H} 1}{ }^{0}$ are the concentration of $\mathrm{H} 1, \mathrm{H} 2$, HR and H 1 before reaction respectively. Similarly $p_{\mathrm{EP}}$ was defined as ( $C_{\mathrm{EP}} / C_{\mathrm{EP}}{ }^{0}$ ), where $C_{E P}$ and $C_{E P}{ }^{0}$ are the concentration before and after the reaction starts. Note that practically $p_{\mathrm{HR}}$ is equivalent with the extent of reaction measured by FT-IR method.[1]
(2) As the reaction proceeds the state of PPG diamine unit varies. The state was classified into six as shown schematically in Figure 1.
(3) Using the probability, $\gamma_{a}$ and $\gamma_{b}$, the equation for gelation was written as $\gamma_{a} \times \gamma_{b}=1$ (The definition of $\gamma_{a}$ and $\gamma_{b}$ was written in detail below.), where ring forming parameters were not incorporated. The result derived from this equation was that $p_{\mathrm{HR}}=(1 / \sqrt{3})$. It is
consistent with the gel point of the classical Flory-Stockmayer theory, which proves the validity of the molecular model shown in previous study.
As a further consideration to these remarks, this study discusses the network formation model including the ring forming parameter.

## 2. PROBABILITY OF PATH CONTINUATION

Concerning the classification for the states of PPG diamine unit shown in Figure 1, $X_{i}$ can be expressed as a function of $p_{\mathrm{H} 1}, p_{\mathrm{H} 2}$ and $p_{\mathrm{HR}}$, where the mole fractions of these six states were written as $X_{i}(i=1 \sim 6) . \quad X_{i}$ finally become functions of $p_{\mathrm{HR}}$, because $p_{\mathrm{H} 1}$ and $p_{\mathrm{H} 2}$ can be given as functions of $p_{\mathrm{HR}}$.
$X_{1}=p_{\mathrm{H} 1}{ }^{2} \quad X_{2}=4 p_{\mathrm{H} 1} p_{\mathrm{H} 2}$
$X_{3}=2 p_{\mathrm{H} 1}\left(p_{\mathrm{HR}}-p_{\mathrm{H} 2}\right), X_{4}=4 p_{\mathrm{H} 2}{ }^{2}$
$X_{5}=4 p_{\mathrm{H} 2}\left(p_{\mathrm{HR}}-p_{\mathrm{H} 2}\right), X_{6}=\left(p_{\mathrm{HR}}-p_{\mathrm{H} 2}\right)^{2} \quad(1-6)$.

| State 1 | State 2 | State 3 |
| :---: | :---: | :---: |
|  |  |  |
| State 4 | State 5 | State 6 |
|  |  |  |

Figure 1. Classification of PPG diamine unit.
In the network model without ring forming parameter the probabilities of $Q_{\mathrm{H},}, Q_{\mathrm{H} 2}$ and $Q_{\mathrm{HR}}$ were introduced to express those of finding $\mathrm{H} 1, \mathrm{H} 2$ and HR in PPG diamine unit; some examples of them were shown below.
$Q_{\mathrm{H}, i=1}=\frac{4 X_{1}}{4 X_{1}+2 X_{2}+2 X_{3}}, \quad Q_{\mathrm{H} 1, i=4}=0$
$Q_{\mathrm{H} 1, i=2}=\frac{2 X_{2}}{4 X_{1}+2 X_{2}+2 X_{3}}, \quad Q_{\mathrm{H} 1, i=5}=0$,
$Q_{\mathrm{H} 1, i=3}=\frac{2 X_{3}}{4 X_{1}+2 X_{2}+2 X_{3}}, Q_{\mathrm{H} 1, i=6}=0 .(7-12)$
Furthermore, in the PPG diamine unit, $\beta_{\mathrm{i}}$, the number of way out to go from a reacted H atom to a next diepoxy unit was counted, so that the probabilities, $\gamma_{\mathrm{a}}$ and $\gamma_{\mathrm{b}}$ could be given. $\quad \gamma_{a}$ stands for the probability of path
continuation out of PPG diamine unit attached to a randomly chosen H atom, either reacted or unreacted. Similarly, $\gamma_{b}$ stands for the probability out of DGEBA unit attached to a randomly chosen epoxy group, either reacted or unreacted. (These descriptions will appear again in Section 3.)

There are three kinds of contributions to $\gamma_{\mathrm{a}}$; namely, those come from $Q_{\mathrm{H} 1}, Q_{\mathrm{H} 2}$ and $Q_{\mathrm{HR}}$. If we regard $S_{\mathrm{H} 1}$ as the contribution from H 1 to $\gamma_{\mathrm{a}}$, then $S_{\mathrm{H} 1}$ means the probability of path continuation from H 1 atom to the next DGEBA unit on condition that randomly chosen H atom was H1 atom. Similar definitions can be given for the probabilities $S_{\mathrm{H} 2}$ and $S_{\mathrm{HR}}$. Therefore we have,

$$
\gamma_{\mathrm{a}}=S_{\mathrm{H} 1}+S_{\mathrm{H} 2}+S_{\mathrm{HR}}
$$

On the other hand, the probability to chose H 1 atom from all of H atoms is $p_{\mathrm{H} 1}$ which can be determined by the experiment. Now, $S_{\mathrm{Hl}_{6}}$ can be obtained as follows;

$$
\begin{equation*}
S_{\mathrm{H} 1}=p_{\mathrm{H} 1} \times \sum_{i=1}^{6} Q_{\mathrm{H} 1, i} \times \beta_{i} \tag{14}
\end{equation*}
$$

where $\beta_{1}=\beta_{4}=\beta_{5}=\beta_{6}=0, \beta_{2}=1, \beta_{3}=2$. These $\beta_{i}$ values can be determined by counting the number of H1 and HR atoms.

Similarly, the expressions for $Q_{\mathrm{H} 2, i}$ and $\beta_{i}$, as well as those for $Q_{\mathrm{HR}, i}$ and $\beta_{i}(i=1 \sim 6)$ can be written using $X_{i}$ Therefore, $\gamma_{\mathrm{a}}$ can be calculated. Furthermore, $\gamma_{\mathrm{b}}$ can be calculated as a function of $p_{\mathrm{HR}}$, consequently $\gamma_{\mathrm{b}}=p_{\mathrm{HR}}$ holds.

## 3. RING FORMING PARAMETER

In order to modify classical gelation theory, the ring forming parameter was introduced into the network formation model used in this study. The ring forming is illustrated in Figure 2 to show the competition between inter- and intramolecular reaction occurring throughout a polymerisation.[8-10] In the illustration, A means a H atom in PPG diamine, B is an epoxy group. The ta group is about to react with an epoxy group. $\quad C_{b, \text { int }}$ is the total effective concentration of B groups from the same molecule around 因. $C_{\mathrm{b}, \text { ext }}$ is the concentration of B groups from other molecules. $\quad v$ is the number of bonds in the chain forming the smallest ring structure.

It is assumed first that the intrinsic rate of reaction between A and B group is insensitive to whether inter- or intramolecular reaction is occurring, and second that the probabilities of inter- and intramolecular reactions are directly proportional to concentrations. The first assumption neglects any orientation effects which may be prevalent in intramolecular reaction, and the second neglects the fact that the reaction between the amine and the epoxy groups may have a complex mechanism and the expression for the rate of reaction may not be simply factorisable in terms of concentrations.

The ring forming parameter $\lambda_{b}$ is given concerning B group as;

$$
\begin{equation*}
\lambda_{\mathrm{b}}=\frac{C_{\mathrm{b}, \text { int }}}{C_{\mathrm{b}, \mathrm{int}}+C_{\mathrm{b}, \mathrm{ext}}} \tag{15}
\end{equation*}
$$

Similarly, $\lambda_{\mathrm{a}}, C_{\mathrm{a}, \mathrm{jn}}$, and $C_{\mathrm{a}, \text { ext }}$ can be defined when a B group is about to react with A groups.

As a next step, the size of ring structure was given; for example, the smallest ring structure is comprised of a PPG diamine and a DGEBA unit. This structure is regarded as that of $j=1$. The second smallest ring consists of two of each unit and was regarded as the
structure of $j=2$. In consequence, $C_{b, \text { int }}$ shown in Figure 2 is the total of the concentration of each size.


Figure 2. Schematic representation of the competition between inter- and intramolecular reaction. See text in detail.

$$
\begin{equation*}
C_{\mathrm{b}, \text { int }}=\sum_{j=1}^{\infty} C_{\mathrm{b}, \text { int }, j} \tag{16}
\end{equation*}
$$

In counting the concentration $\mathcal{C}_{\mathrm{b}, \text { int, }, j}$, a specific feature was given below; that is, the ring structure can be regarded as a chain which end-to-end distance is equal to zero.[11] It is then, assumed that the distribution of end-to-end distance can be written by independent Gaussian sub-chain statistics of eq.(17) for the polymer chain including PPG diamine and DGEBA.

$$
\begin{align*}
& P(\vec{r}, j v)=\sqrt{\left(\frac{3}{2 \pi<r^{2}>}\right)^{3}} \exp \left(-\frac{3 r^{2}}{\left.2<r^{2}\right\rangle}\right)  \tag{17}\\
& \left\langle r^{2}\right\rangle=v j b^{2}
\end{align*}
$$

$\bar{r}$ is end-to-end vector. $b$ is the effective bond length of the chain of $v j$ bonds. In particular, if a definition is given on $P_{\text {ab }}$ with dividing by Avogadro constant, $N_{\mathrm{av}}$ and substituting $\vec{r}=0$ and $j=1$, then it is a useful parameter in counting the concentration of $C_{\mathrm{a}, \mathrm{int}, j}$ and $C_{\mathrm{b}, \mathrm{int}, j}$ :

$$
\begin{equation*}
P_{\mathrm{ab}}=\frac{1}{N_{\mathrm{AV}}} \sqrt{\left(\frac{3}{2 \pi<r^{2}>}\right)^{3}} \tag{19}
\end{equation*}
$$

Once the molecular structure of the monomer is given, values of $v$ and $b$ can in principle be estimated. $\quad P_{\mathrm{ab}}$ means the concentration of intramolecular B groups of $j=1$ around the A group; it can also be said that $P_{\mathrm{ab}}$ means the concentration of intramolecular A groups of $j=1$ around the $B$ group. Equation (19) neglects any effects of the connectivity of the branch units on the statistics governing the mutual separations of pairs of end groups in a branched chain. Consequently, eq.(16) can be calculated as follows;
$C_{\mathrm{b}, \text { int }}=$
$\sum_{j=1}^{\infty} \frac{1}{N_{\mathrm{AV}}} \sqrt{\left(\frac{3}{2 \pi<r^{2}>}\right)^{3}}=\frac{1}{N_{\mathrm{AV}}} \sum_{j=1}^{\infty} \sqrt{\left(\frac{3}{2 \pi j b^{2}}\right)^{3}}=P_{\mathrm{ab}} \sum_{j=1}^{\infty}\left(\frac{1}{j}\right)^{3 / 2}=2.61 P_{\mathrm{ab}}$
In the above calculation Truesdell function was taken into account.[12] Its use allows ring structures of all sizes ( $\mathrm{j}=2,3,---$ ) to be accounted for. With these preparations, now $\lambda_{\mathrm{a}}$ and $\lambda_{\mathrm{b}}$ become solvable. The outline to find $P_{\mathrm{ab}}$, which is the final target in our research work, is; first, to give ( $\gamma_{a} \times \gamma_{b}$ ) as a
function of $P_{\mathrm{ab}}$ and $p_{\mathrm{HR}}$., then put $p_{\mathrm{HR}}$ value at gel point obtained from experiments to the equation of gelation, $\left(\gamma_{a} \times \gamma_{b}\right)=1$.


Figure 3. Schematic representation for the chain growth in the polymerisation of DGEBA and PPG diamine.

It should be noted for $\gamma_{\mathrm{a}}$ and $\gamma_{\mathrm{b}}$ using the schematic representation of Figure 3 for the chain growth in a polymerisation. Imagine that the chain grows from right to left, specifically the growth from $\mathrm{B}^{1}$ to $\mathrm{B}^{2}$ was considered in the following sections. $\gamma_{b}$ is the probability that $B^{1}$ reacts with PPG diamine of the left side, $\gamma_{a}$ is the probability that $A^{1}$ reacts with DGEBA of left side.

## 4. INTERNAL CONCENTRATION OF H ATOM

In order to count the number of $A$ group around $\mathrm{B}^{1}$, the H atom reacted with (B) which is paired with $\mathrm{B}^{1}$ was considered, then $C_{\mathrm{a}, \mathrm{int}, j}$ became countable. The probability that the PPG diamine is in $i$-th $\operatorname{state}(i=1 \sim 6)$ is equivalent to $Q_{\mathrm{HR}, i}$, because $Q_{\mathrm{HR}, i}$ was the fraction of $i$-th state on condition that the reacted H atom had been chosen. Obviously we have the following equation.
$Q_{\mathrm{HR}, i=2}+Q_{\mathrm{HR}, i=3}+Q_{\mathrm{HR}, i=4}+Q_{\mathrm{HR}, i=5}+Q_{\mathrm{HR}, i=6}=1$
The number of the intramolecular A groups around $\mathrm{B}^{1}$ was counted in order of $j$ when PPG diamine was in $i$-th state in the estimation of $C_{\text {a,int }, j}$. Note that $C_{\mathrm{a}, \mathrm{int}, j}$ was written hereafter with abbreviating the subscript of $i$ to make it simple. It can be expressed as;
[intramolecular A groups]
$=Q_{\mathrm{HR}, i} C_{\mathrm{a}, \mathrm{int} . j} \times$ [unreacted A groups $]$
Square brackets mean the concentration. Estimation of $C_{\mathrm{a}, \text { int }, i}$ was described below concerning $i=1 \sim 6$ on the basis of this equation.
(1) $C_{\text {a, inti,i=2 }}$ : (B) reacts with the $2^{\text {nd }}$ state.
$\mathbf{B}^{1}$ forms a ring of $j=1$. Because the forming has three ways, the concentration of A groups of $j=1$ is $C^{0}{ }_{H 1} \times$ $p_{\mathrm{EP}} Q_{\mathrm{HR}, i=2} C_{\mathrm{a}, \mathrm{in}, j, j=1} \times 3$; where, $p_{\mathrm{EP}}$ comes from the condition that the reaction of $(B)$ is necessary for PPG diamine to be the $2^{\text {nd }}$ state.

The ring formation of $j>2$ is impossible when the PPG diamine is in the $2^{\text {nd }}$ state. Therefore we have the following equation.

$$
C_{\text {a, int }, i=2}=C_{H 1}^{0} \times p_{\mathrm{EP}} Q_{\mathrm{HR}, i=2} C_{\text {a, int }, j=1} \times 3
$$

$C_{\mathrm{a}, \mathrm{int}, j=1}=P_{\mathrm{ab}}$ holds when there are no chains of $j>2$. Moreover, we have $p_{\mathrm{EP}}=p_{\mathrm{HR}}$ when the stoichiometric ratio is unity. Therefore, the internal concentration can be given as;

$$
\begin{equation*}
C_{\mathrm{a}, \mathrm{int}, i=2}=C_{\mathrm{H} 1}^{0} \times 3 p_{\mathrm{HR}} Q_{\mathrm{HR}, i=2} P_{\mathrm{ab}} . \tag{23}
\end{equation*}
$$

(2) $C_{\text {a, int }, i=3}:(\mathrm{B})$ reacts with the $3^{\text {rd }}$ state.
$\mathrm{B}^{1}$ forms a ring of $j=1$. Because the forming has two ways, the concentration of A group of $j=1$ is $C^{0}{ }_{\mathrm{H} 1} \times p_{\mathrm{EP}} Q_{\mathrm{HR}, i=3} C_{\mathrm{a}, \mathrm{in}, j, j=1} \times 2$. $p_{\mathrm{EP}}$ comes from the same reason as the situation of $i=2$. The concentration of $j=2$ is $C^{0}{ }_{\mathrm{H} 1} \times p_{\mathrm{EP}} Q_{\mathrm{HR}, i=3} C_{\text {a, int }, j=2} \times 2\left(1-p_{\mathrm{HR}}\right)$. The factor 2(1$\left.p_{\mathrm{HR}}\right)$ is the number of opportunities of forming a ring structure, that is, the number of unreacted A groups on

PPG diamine unit of $j=2$. The same expressions can be obtained for the concentration of $j=3,4,---$. The total internal concentration on condition that the reacted PPG diamine was in the $3^{\text {rd }}$ state can be obtained from the sum over $j=2,3,4, \cdots$.
$C_{\mathrm{a}, \mathrm{int}, i=3}=C_{\mathrm{H} 1}^{0} \times 2 p_{\mathrm{HR}} Q_{\mathrm{HR}, i=3} P_{\mathrm{ab}}\left(2+3.22\left(1-p_{\mathrm{HR}}\right)\right)$
(3) $C_{a, \text { int } i=4}:(B)$ reacts with the $4^{\text {th }}$ state.
$B^{1}$ forms a ring of the same structure as that considered in $C_{\mathrm{a}, \text { int }, i=3}$. Therefore $C_{\mathrm{a}, \mathrm{in}, t, i=4}$ can be given by the following equation.

$$
\begin{equation*}
C_{\mathrm{a}, \text { int }, i=4}=C_{\mathrm{H} 1}^{0} \times 2 p_{\mathrm{HR}} Q_{\mathrm{HR}, i=4} P_{\mathrm{ab}}\left(2+3.22\left(1-p_{\mathrm{HR}}\right)\right) \tag{25}
\end{equation*}
$$

(4) $C_{\text {a, inti } i=5}:(\mathrm{B})$ reacts with the $5^{\text {th }}$ state.
$\mathrm{B}^{1}$ forms a ring of $j=1$ in only one way. The concentration of A group of $j=1$ is $C^{0}{ }_{\mathrm{H} 1} \times p_{\mathrm{EP}} Q_{\mathrm{HR}, i=5} C_{\mathrm{a}, \text { int } j, j}$. The concentration of $j>2$ can be derived in the same way as that of $C_{a, i n t, i=3}$. Because there is bifurcation it becomes twice as large as the concentration when (B) reacts with $3^{\text {rd }}$ state. Thus, the following can be given.
$C_{\mathrm{a}, \mathrm{int}, i=5}=p_{\mathrm{EP}} Q_{\mathrm{HR}, i=5} C_{\mathrm{a}, \text { in } \mathrm{n}, j=1}+2 \sum_{j=2}^{\infty} p_{\mathrm{EP}} Q_{\mathrm{HR}, i=5} C_{a, \text { in }, j} \times 2\left(1-p_{\mathrm{HR}}\right)$
$=C^{0}{ }_{\mathrm{HI}} \times p_{\mathrm{EP}} Q_{\mathrm{HR}, i=5} P_{\mathrm{ab}}\left[1+4(\phi(1,3 / 2)-1)\left(1-p_{\mathrm{HR}}\right)\right]$
$=C_{\mathrm{H} 1}^{0} \times P_{\mathrm{HR}} Q_{\mathrm{HR}, i=5} P_{\mathrm{ab}}\left[1+6.44\left(1-p_{\mathrm{HR}}\right)\right]$
(5) $C_{\mathrm{a}, \text { int }, i=6}:(\mathrm{B})$ reacts with the $6^{\text {th }}$ state.

When (B) reacts with PPG diamine of the $6^{\text {th }}$ state the ring structure of $j=1$ does not form. Concerning the ring of $j=2$, the derivation used when $C_{\mathrm{a}, \mathrm{int}, i=3}$ was counted can be applied to give $C_{\mathrm{a}, \text { int }, j=2}$. The $6^{\text {th }}$ state causes three-ways branching in comparison with the $3^{\text {rd }}$ state.
$C_{\mathrm{a}, \mathrm{int}, i=6}=C_{\mathrm{H} 1}^{0} \times 3 \sum_{j=2}^{\infty} p_{\mathrm{EP}} Q_{\mathrm{HR}, i=6} C_{\mathrm{a}, \mathrm{int}, j} \times 2\left(1-p_{\mathrm{HR}}\right)$
$=C^{0}{ }_{\mathrm{H} 1} \times 6 p_{\mathrm{EP}} Q_{\mathrm{HR}, i=6} P_{\mathrm{ab}}(\phi(1,3 / 2)-1)\left(1-p_{\mathrm{HR}}\right)$
$=C_{\mathrm{H} 1}^{0} \times 9.66 p_{\mathrm{HR}} Q_{\mathrm{HR}, i=0} P_{\mathrm{ab}}\left(1-p_{\mathrm{HR}}\right)$
As shown in equations (23-27), $C_{\mathrm{a}, \mathrm{int}, i}$ could be counted for $i=1 \sim 6$. It should be noted that $C_{\mathrm{a}, \mathrm{int}, i}$ is expressed by three variables of $p_{\mathrm{HR}}, Q_{\mathrm{HR}, i}$ and $P_{\mathrm{ab}}$. Further, the value of $Q_{\mathrm{HR}, i}$ is fixed if $p_{\mathrm{HR}}$ is determined. Consequently the expression of $\lambda_{a}$ can be written using $p_{\mathrm{HR}}$ and $P_{\mathrm{ab}}$.

## 5. INTERNAL CONCENTRATION OF EPOXY GROUP

Next, the probability of the intramolecular reaction is calculated when $A^{1}$ is on the point of reacting with DGEBA of left side as shown in Figure 3. That is, we are going to count $C_{b, i n t, i}$ with the manner used in counting $C_{\mathrm{a}, \text { int }, i}$. It follows that, at first a PPG diamine unit including $A^{1}$ is classified according to $i=1 \sim 6$, then the internal concentration of epoxy group is counted for $j$ using the same idea of eq. (22).

It obviously cannot happen that the PPG diamine is in the $1^{\text {st }}$ state because the PPG diamine unit is presupposed to react with $B^{1}$. In addition, ring formation is impossible when the PPG diamine is in the $2^{\text {nd }}$ state. Thus, $C_{\mathrm{b}, \text { int }, i=1,2}=0$.
(1) $C_{\mathrm{b}, \text { int }, i=3}: \mathrm{A}^{1}$ is included in the $3^{\mathrm{rd}}$ state.

Specific reactions of the PPG diamine in the $3^{\text {rd }}$ state were considered. $\quad A^{1}$ forms a ring structure of $j=1$ only when ( $B$ ) is unreacted. The concentration of (B) is $C^{0}{ }_{\mathrm{H} 1} \times Q_{\mathrm{HR}, i=3} \times C_{\mathrm{b}, \mathrm{int}, j=1}\left(1-p_{\mathrm{EP}}\right)$.

Concerning the ring structure of $j=2$, the internal concentration of B can be written as $C^{0}{ }_{\mathrm{H}} \times Q_{\mathrm{HR}, i=3} \times$ $p_{\mathrm{EP}} \times C_{\mathrm{b}, \mathrm{int}, j=2} \times 2 p_{\mathrm{HR}}\left(1-p_{\mathrm{EP}}\right)$. $\quad p_{\mathrm{EP}}$ preceding $C_{\mathrm{b}, \text { int }, j=2}$ was placed because the reaction of $(\mathrm{B})$ is necessary to become a ring of $j=2$. $\quad p_{\mathrm{HR}}$ was placed because the reaction of PPG diamine at the right side of $(\mathrm{B})$ is necessary to become a ring of $j=2$. The concentration of B groups of $j>3$ can be counted similarly. The result of addition over $j=1,2,3$, --- will be given below.
$C_{\mathrm{b}, \mathrm{in}, i=3}=C_{\mathrm{H} 1}^{0} \times Q_{\mathrm{HR}, i=3} \times C_{\mathrm{b}, \mathrm{int}, j=1}\left(1-p_{\mathrm{EP}}\right)$

$$
\begin{align*}
& \quad+C_{\mathrm{H} 1}^{0} \times \sum_{j=2}^{\infty} Q_{\mathrm{HR}, i=3} \times p_{\mathrm{EP}} C_{\mathrm{b}, \mathrm{int}, j} \times 2 p_{\mathrm{HR}}\left(1-p_{\mathrm{EP}}\right) \\
& =C_{\mathrm{H} 1}^{0} \times Q_{\mathrm{HR}, i=3} P_{\mathrm{ab}}\left(1-p_{\mathrm{HR}}\right)\left(1+3.22 p_{\mathrm{HR}}{ }^{2}\right) \tag{28}
\end{align*}
$$

(2) $C_{\mathrm{b}, \text { int }, i=4}: \mathrm{A}^{1}$ is included in the $4^{\text {th }}$ state.
$A^{1}$ forms a ring of the same structure as that considered in $C_{\mathrm{b}, \text { int }, i=3}$. Therefore $C_{\mathrm{b}, \text { int }, i=4}$ can be given by the following equation.

$$
\begin{equation*}
C_{\mathrm{b}, i \mathrm{int}, i=4}=C_{\mathrm{H} 1}^{0} \times Q_{\mathrm{HR}, i=4} P_{\mathrm{ab}}\left(1-p_{\mathrm{HR}}\right)\left(1+3.22 p_{\mathrm{HR}}^{2}\right) \tag{29}
\end{equation*}
$$

(3) $C_{b, \text { int }, i=5}: A^{1}$ is included in the $5^{\text {th }}$ state.

To divide the situation into two whether (B) is reacted or unreacted will help to consider the concentration of intramolecular $B$ group. If ( $B$ ) is unreacted the ring structure will be $j=1$, which forms in two ways. Thus the concentration is $C^{0}{ }_{\mathrm{H} 1} \times 2 Q_{\mathrm{HR}, i=5} \times C_{\mathrm{b}, \text { int }, j=1}\left(1-p_{\mathrm{EP}}\right)$. If (B) is reacted then the ring structure will be that of $j>2$, and there are two ways to form it. For $j=2$, the concentration is $C_{\mathrm{H} 1}^{0} \times Q_{\mathrm{HR}, i=5} \times p_{\mathrm{EP}} C_{\mathrm{b}, \text { int }, j=2} \times 2 p_{\mathrm{HR}}\left(1-p_{\mathrm{EP}}\right)$ in each way, then the total is double the concentration. Similar expression can be obtained for $j=3,4--; C_{\mathrm{b}, \text { int }, i=5}$ can be given by the sum over $j=1,2,3 \cdots$.
$C_{\mathrm{b}, \mathrm{int}, i=5}=C^{0}{ }_{\mathrm{H} 1} \times$
$2 Q_{\mathrm{HR}, i=5} C_{\mathrm{b}, \mathrm{in} 1, j=1} \times\left(1-p_{\mathrm{EP}}\right)+2 \sum_{j=2}^{\infty} Q_{\mathrm{HR}, i=5} p_{\mathrm{EP}} C_{\mathrm{b}, \mathrm{int}, j} \times 2 p_{\mathrm{HR}}\left(1-p_{\mathrm{EP}}\right)$
$=C^{0}{ }_{\mathrm{H} 1} \times 2 Q_{\mathrm{HR}, i=5} \times P_{\mathrm{ab}}\left(1-p_{\mathrm{HR}}\right)\left(1+3.22 p_{\mathrm{HR}}{ }^{2}\right)$
(4) $C_{\mathrm{b}, \mathrm{jnt}, i=6}: \mathrm{A}^{1}$ is included in the $6^{\mathrm{h}}$ state.

If ( B ) is unreacted, the ring of $j=1$ forms in three ways. Thus, the internal concentration of $B$ around $A^{1}$ is $C^{0}{ }_{\mathrm{H} 1} \times 3 Q_{\mathrm{HR}, i=6} \times C_{\mathrm{b}, \mathrm{int}, j=1}\left(1-p_{\mathrm{EP}}\right)$. If $(\mathrm{B})$ is reacted then it forms the ring structure of $j>2$ in three ways. As a consequence;

$$
\begin{align*}
& C_{\mathrm{b}, \text { int }, i=6}=C_{\mathrm{H} 1}^{0} \times 3 Q_{\mathrm{HR}, i=6} \times C_{\mathrm{b}, \mathrm{int}, j=1} \times\left(1-p_{\mathrm{EP}}\right)+ \\
& C_{\mathrm{H} 1 \times 3}^{0} \sum_{j=2}^{\infty} Q_{\mathrm{HR}, i=6} p_{\mathrm{EP}} C_{\mathrm{a}, \mathrm{im}, j} \times 2 p_{\mathrm{HR}}\left(1-p_{\mathrm{EP}}\right) \tag{31}
\end{align*}
$$

$=C_{\mathrm{H} 1}^{0} \times 3 Q_{\mathrm{HR}, i=6} \times P_{\mathrm{ab}}\left(1-p_{\mathrm{HR}}\right)\left(1+3.22 p_{\mathrm{HR}}^{2}\right)$
As shown in equations $(28-31), C_{b, \text { int }, i}$ could be counted for $i=1 \sim 6$.

## 6. THE INCORPORATION OF RING FORMING PARAMETER INTO $\gamma_{\mathrm{a}}$ and $\gamma_{\mathrm{b}}$

As can be seen in Section 4, $C_{\mathrm{a}, \mathrm{int}, i}$ could be counted. It is necessary to show the relation between $C_{\mathrm{a}, \mathrm{in}, i}$ and $\lambda_{\mathrm{a}}$ as well as $\lambda_{a}$ and $\gamma_{b}$. It is also necessary to show the relation of $C_{b, i n b i}, \lambda_{\mathrm{b}}$ and $\gamma_{\mathrm{a}}$ in order to accomplish the equation for network formation including the ring forming parameter.

Then, $\lambda_{a}$ ' was defined as shown in the following equation, to give the relation between $C_{\mathrm{a}, \text { int } i}$ and $\lambda_{\mathrm{a}}$.
$\lambda_{\mathrm{a}}{ }^{3}=C_{\mathrm{a}, \mathrm{int}} / C_{\mathrm{a}, \mathrm{cxt}}=\sum_{i=1}^{6} C_{\mathrm{a}, \mathrm{int}, i} / C_{\mathrm{Ht}}^{0}\left(1-p_{\mathrm{HR}}\right)$

The relation between $\lambda_{\mathrm{a}}$ and $\lambda_{\mathrm{a}}{ }^{\prime}$ is;

$$
\lambda_{\mathrm{a}}=\left(\lambda_{\mathrm{a}}^{\prime} / 1+\lambda_{\mathrm{a}}^{\prime}\right)
$$

The calculation of $\gamma_{b}$ was mentioned briefly in Section 2 ; the number of way out must be counted in this calculation. The number of way out to continue to next PPG diamine through the intermolecular reaction is $\left(1-\lambda_{a}\right)$ taking account of the ring formation. Therefore the expression for $\gamma_{b}$ can be obtained as; [6]
$\gamma_{\mathrm{b}}=\left(1-p_{\mathrm{EP}}\right) \frac{Z_{2}}{2 Z_{1}+Z_{2}}\left(1-\lambda_{\mathrm{a}}\right)+p_{\mathrm{EP}} \frac{2 Z_{3}}{Z_{2}+2 Z_{3}}\left(1-\lambda_{\mathrm{a}}\right)$
On the other hand, $\lambda_{\mathrm{b}, i}$ can be expressed using $C_{\mathrm{b}, \text { int }, i}$ derived in Section 6 for $i=1 \sim 6$ as;

$$
\begin{equation*}
\lambda_{\mathrm{b} i}=C_{\mathrm{b}, \mathrm{int}, i} /\left(C_{\mathrm{b}, \mathrm{int}, i}+C_{\mathrm{b}, \mathrm{ex}}\right) \tag{35}
\end{equation*}
$$

The same idea as used in eq. (34) can be applied for $\lambda_{b i}$ in incorporating it into $\gamma_{a}$. Thus;

$$
\begin{array}{r}
\gamma_{\mathrm{a}}=p_{\mathrm{H} 1} \times \sum_{\mathrm{i}=1}^{6} Q_{\mathrm{H}, \mathrm{i}} \times \beta_{i}\left(1-\lambda_{\mathrm{bi}}\right)+p_{\mathrm{H} 2} \times \sum_{i=1}^{6} Q_{\mathrm{H} 2, \mathrm{i}} \times \beta_{\mathrm{i}}\left(1-\lambda_{\mathrm{bi}}\right)^{+} \\
p_{\mathrm{HR}} \times \sum_{i=1}^{6} Q_{\mathrm{HR}, \mathrm{i}} \times \beta_{\mathrm{i}}\left(1-\lambda_{\mathrm{bi}}\right) \tag{36}
\end{array}
$$

Although eqs. (34) and (36) appear to include a lot of variables, in fact, they comprise only two variables of $p_{\mathrm{HR}}$ and $P_{\mathrm{ab}}$. Therefore, if $p_{\mathrm{HR}}$ was given at the gelation point from the experimental result, $P_{\mathrm{ab}}$ can be obtained.

## 7. CONCLUDING REMARKS

Gelation process of the cure of DGEBA and PPG diamine could be analysed with the extent of reaction of $p_{\mathrm{HR}}$. A molecular model including the ring forming parameter was constructed to be applied for the gelation process. The classification of PPG diamine was carried out according to the reaction state; that was crucial in incorporating the ring forming parameter into the model.

The comparison of $p_{\mathrm{HR}}$ values between those determined by this theory and measured experimentally is now under the process of carrying out.

## REFERENCES

[1] Y. Tanaka, Polym. Prep. 53, (2) 3265(2004); ibid, 53, (1), 957(2004)
[2] K. Dusek, American Chem. Soc. Symp. Ser., 208, 3-14(1984)
[3] J. P. Eloundou, M. Feve, D. Harran, J. P. Pascault, Ang. Makromol. Chem., 230, 13-46(1995)
[4] R. B. Prime, Thermal Characteristics of Polymeric Materials, Chap.5., E. Turi, Ed., Academic Press, New York, (1982)
[5] J. P. Eloundou, M. Feve, J. F. Gerard, D. Harran, J. P. Pascault, Macromolecules, 29, 6907-6916(1996); J. P. Eloundou, J. F. Gerard, D. Harran, J. P. Pascault, ibid, 6917-6927(1996)
[6] Y. Tanaka, Trans. Mat. Res. Soc. Jpn., 30[3], 819-822(2005)
[7] M. Gordon and S.B. Ross-Murphy, Pure Appl. Chem., 43, 1-26(1975)
[8] Z. Ahmad and R. F. T. Stepto, Coll. Polym. Sci., 258, 663-674(1980)
[9] P. J. Flory, A. J. Semlyen, J. Am. Chem. Soc., 88, 3209(1966)
[10] J. N. Greenshields, R. H. Peters, R. F. T. Stepto, J. Chem. Soc., 5101(1964)
[11] H.Jacobson, W.H.Stockmayer, J. Chem. Phys., 18(12)1600-1606(1950)
[12] C. Truesdell, An. Math., 46(1), 144-157 (1945)

