# The origin of anomalous transport property of Al-based quasicrystals studied by positron annihilation

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We have presented one of mechanisms of anomalous transport properties of stable Al-based quasicrystals, taking into account experimental date for stable Al-based quasicrystals and those approximant crystals by positron annihilation methods.

Key words: quasicrystal, transport, positron annihilation, clusters, pseudogap

# 1. INTORODUCTION

Stable icosahedral quasicrystals of multinary aluminium-transition metal alloys are known to exhibit semiconductor-like and even insulator-like electronic transport properties [1]. It has been suggested that the semiconductor-like properties might be interpreted by the combination of a pseudogap in the electron density of states at the Fermi level and localization tendency of electrons near the Fermi energy  $E_{\rm F}$  [2]. The origin of the pseudogap at  $E_{\rm F}$  in quasicrystals might be ascribed to a Hume-Rothery mechanism, namely the scattering of Fermi electrons by planes of a "Brillouin" zone constructed from the most intense peaks of the diffraction diagram. Janet and de Boissien [3] proposed a cluster model to explain the properties and the stability of quasicrystals. The icosahedral structure is based upon clusters that contain a number of electrons such that it corresponds to one of the magic number which stabilize the structure of atomic aggregates. In the cluster of N atoms with n electrons per atom, a special situation arises if the total number of electrons N  $\cdot$  n differ from a magic number M by n. The cluster would be slightly reactive, and it will have a tendency to grow into a super cluster of N clusters. Mayou et al. [4] proposed a possible explanation for the unusual transport properties of quasicrystals in terms of hopping processes between wave functions mainly localized inside atomic clusters. Kimura et al. [5] have discussed the importance of vacant centers of the icosahedral clusters in the anomalous properties and stabilities of Al-based quasicrystals. That is, there are two types of icosahedral clusters, that with thirteen Al atoms (with one atom inter center) and that with twelve Al atoms (with the vacant center). Packing fraction of thirteen-atoms icosahedron is 0.726, which is near to 0.744, that of thirteen atoms close packed structure, and is greater than 0.680, that of body centered cubic structure. The thirteen Al atoms icosahedron must be stable under metallic banding and may be called to be a metallic cluster. On the other hand, packing fraction of the twelve Al atoms icosahedron is 0.640, which is much smaller than those of metallic clusters and approaches to 0.340, that of the diamond structure. The twelve Al atoms icosahedron can not be stable under metallic bonding. This situation shows that the thirteen A1 atoms icosahedron is metallic and the

twelve Al atoms icosahedron has a covalent bouding nature. Through aggregation of Al<sub>12</sub> icosahedral clusters with vacant centers, the valence electron state in  $Al_{12}$ icosahedral cluster induces the valence band, in which the electron kinetic energy becomes lower, and the pseudogap appears near Fermi energy as shown in Fig. 1. Electrical conductivities of Al-based icosahedral quasicrystals are extremely low as compared with those of usual metals because of the pseudogap and the localization tendency of electrons near the Fermi level. Thus it is of significance to study the relationship among the existence of structural vacancies, the bonding nature, and the anomalous transport properties in Al-based quasicrystals and those approximant crystals. Positron annihilation is the most powerful method for detecting vacancy-type defects and structural vacancies in the quasicrystalline and amorphous alloys because their detection is independent on the structural periodicity unlike the diffraction method.

In this report, we have discussed the origin of anomalous transport properties of stable Al-based quasicrystals, taking into account positron annihilation data for Al-based quasicrystals and those approximant crystals.



Fig.1 Formation of a pseudogap

### 2. THE STRUCTURAL VACANCIES OF STABLE AI-BASED QUASICRYSTALS AND APPROXIMANT CRYSTALS

Table 1. show positron annihilation lifetimes of stable Al-based quasicrystals and approximant crystals, which were measured by Kanazawa and co-workers. The lifetime  $\tau_1$  is in the range  $205 \pm 4$  psec from room temperature to 350°C in the quasicrystal Al-Li-Cu [6]. In view of the value  $\tau_1=205\pm4$  psec, it is considered that this component corresponds to that structural vacancies. The lifetime  $\tau_1$  in 1/1-Al-Li-Cu is ~205±4 psec [6]. X-ray analysis of the structure of the 1/1-Al-Li-Cu reveals that the center site of the triacontahedral clusters is vacant. Thus these results suggest that the Al-Li-Cu icosahedral phase also contains a similar density of the same triacontahedral cluster with vacant center as in the 1/1-Al-Li-Cu. Like semiconductors, the resistivity of icosahedral quasicrystal Al-Li-Cu decreases as temperature increases [7]. The lifetimes  $\tau_1$  of quasicrystals AlCuRu and AlCuFe are  $205\pm4$  and  $201\pm4$  ps, respectively [8]. lifetime  $\tau_1$  of 1/1-Al<sub>58</sub>Cu<sub>31.5</sub>Ru<sub>10.5</sub> The and  $1/1-Al_{68}Cu_7Ru_{17}Si_8$  are  $193 \pm 3$  and  $186 \pm 3$  ps, respectively [9]. In the case of 1/1-Al<sub>68</sub>Cu<sub>7</sub>Ru<sub>17</sub>Si<sub>8</sub>, the atomic structure has beem determined from the power x-ray diffraction spectrum using the Rietveld method

[10]. It belongs to the space group Im3 and contains a total of 144 atoms in its unit cell of the bcc structure. The centers of the Mackay clusters located at the vertices and centers of the unit cell are vacant. In addition, the nearest neighbors of these vacant site are Al atoms. We have observed the chemical environment surrounded by Al or Si atoms at the annihilation sites by coincident Doppler broadening spectroscopy [9]. Furthermore, we have observed a chemical environment of Al atoms that is the same as that in icosahedral quasicrystal Al<sub>62.4</sub>Cu<sub>25.4</sub>Ru1<sub>2.2</sub>, as concluded by the coincident Doppler method with a lifetime of 205 ps, corresponding to the structural vacancy [9]. This suggests the notion that the icosahedral quasicrystal Al<sub>64,2</sub>Cu<sub>25,4</sub>Ru<sub>12,2</sub> has the same vacancy-type sites surrounded by Al atoms as the 1/1-Al<sub>68</sub>Cu<sub>7</sub>Ru<sub>17</sub>Si<sub>8</sub> cubic approximant, and therefore it is thought that the structure of the icosahedral quasicrystal Al-Cu-Ru may be composed of a Mackay icosahedral cluster with a vacant center surrounded by Al atoms. The electrical resistivities of quasicrystals Al-Cu-Ru and Al-Cu-Fe are known to be higher than normal metals. These resistivities decrease with increasing temperature like semiconductors [11-13]. The stable icosahedral phase of very grad structural quality in AlPdMn is recognized to be one of the typical F-type quasicrystals of which the structure can be described in terms of a face- centered six-dimensional hypercubic lattice. The Al<sub>71.4</sub>Pd<sub>20.2</sub>Mn<sub>8.4</sub>, positron-annihilation lifetimes in Al<sub>71.9</sub>Pd<sub>19.7</sub>Mn<sub>8.4</sub>, Al<sub>70.2</sub>Pd<sub>20.3</sub>Mn<sub>9.5</sub> and Al<sub>69.1</sub>Pd<sub>21.5</sub>Mn<sub>9.4</sub> polyquasicrystalline icosahedral quasicrystals were measured. The positron-annihilation lifetimes in grown and plastically deformed single quasicrystals of Al<sub>70.6</sub>Pd<sub>21.1</sub>Mn<sub>8.3</sub> were also measured. In every sample the lifetime spectrum consists of a single component with the lifetime of  $206 \pm 4$  ps as shown in Table 1. The results indicate that icosahedral quasicrystalline

Al-Pd-Mn contains a dense distribution of vacancy-type sites. It is known that the transport properties of icosahedral quasicrystalline AlPdMn are non-metallic [15]. The quasicrystals Al-Pd-Re have the highest resistivities in quasicrystals and have semiconductor-like transport property. The positron annihilation lifetime spectra of the quasicrystals Al<sub>70.7</sub>Pd<sub>21.34</sub>Re<sub>7.96</sub>, Al<sub>73</sub>Pd<sub>18</sub>Re<sub>9</sub>, Al<sub>73</sub>Pd<sub>19</sub>Re<sub>8</sub> Al<sub>72</sub>Pd<sub>20</sub>Re<sub>8</sub>, and Al<sub>72</sub>Pd<sub>21</sub>Mn<sub>7</sub> are composed of a single component. For every samples an equal lifetime of  $215 \pm 3ps$  is observed [16]. The values of those lifetimes show that the annihilation site corresponds to the vacant site. Since the lifetime spectra are composed of a single component, it is thought that most thermalized positrons are trapped in vacant sites in a dense distribution. The positron annihilation lifetime spectra of the quasicrystals Al<sub>71</sub>Pd<sub>20</sub>(Re<sub>1-x</sub> Ru<sub>x</sub>) x=0.4,  $Al_{71}Pd_{20}(Re_{1-x} Ru_x) = 0.55$ ,  $Al_{71}Pd_{20}(Re_{1-x} Ru_x) = 0.7$ , and  $Al_{71}Pd_{20}(Re_{1-x} Ru_x) = 0.85$  are composed of a single component. For every samples an equal lifetime of 210  $\pm$  4ps is observed [17]. It is seen that icosahedral quasicrystals Al-Pd-Re-Ru contain a dense distribution of vacancy-type sites. The quasicrystals Al-Pd-Re-Ru have semiconductor-like transport property [18,19].

Table 1. Positron lifetimes of stable Al-based quasicrystals and approximant crystals

As shown in Table 1, the experimental results by positron annihilation lifetime measurements stress that stable Al-based quasicrystals and their approximant crystals, which have non-metallic transport property, contain a dense distribution of structural vacancies without exception. This is consistent with expectation by

Specimen	T <sub>1</sub>	$I_1$	Reference
•	(ps)	(%)	
i-Al <sub>70.7</sub> Pd <sub>21.34</sub> Re <sub>7.96</sub>	213	100	[16]
i-Al <sub>73</sub> Pd <sub>18</sub> Re <sub>9</sub>	218	100	[16]
i-Al <sub>73</sub> Pd <sub>19</sub> Re <sub>8</sub>	215	100	[16]
i-Al <sub>72</sub> Pd <sub>20</sub> Re <sub>8</sub>	218	100	[16]
i-Al <sub>72</sub> Pd <sub>21</sub> Re <sub>7</sub>	213	100	[16]
$i-Al_{71}Pd_{20}(Re_{1-x}Ru_{x})_{9}$	209	100	[17]
x=0.4			
$i-Al_{71}Pd_{20}(Re_{1-x}Ru_{x})_{9}$	206	100	[17]
x=0.55			
$i-Al_{71}Pd_{20}(Re_{1-x}Ru_{x})_{9}$	213	100	[17]
x=0.7			
$i-Al_{71}Pd_{20}(Re_{1-x}Ru_{x})_{9}$	210	100	[17]
x=0.85			
i-Al <sub>71.4</sub> Pd <sub>20.2</sub> Mn <sub>8.4</sub>	210	100	[14]
i-Al71.9Pd19.7Mn8.4	210	100	[14]
i-Al <sub>70.2</sub> Pd <sub>20.3</sub> Mn <sub>9.5</sub>	206	100	[14]
i-Al <sub>691</sub> Pd <sub>215</sub> Mn <sub>94</sub>	211	100	[14]
2/1-Al71Pd20Ru9	212	100	[14]
1/1-Al <sub>72.5</sub> Mn <sub>17.4</sub> Si <sub>10.1</sub>	204	100	[29]
i-Al <sub>60.9</sub> Li <sub>28.9</sub> Cu <sub>10.2</sub>	200	80	[6]
1/1-Al54.6Li34.0Cu11.4	205	80	[6]
i-Al <sub>624</sub> Cu <sub>254</sub> Fe <sub>122</sub>	201	90	[8]
i-Al <sub>63.6</sub> Cu <sub>23.1</sub> Ru <sub>13.3</sub>	205	100	[8]
1/1-Al <sub>58</sub> Cu <sub>31.5</sub> Ru <sub>10.5</sub>	193		[9]
1/0-Al <sub>55</sub> Cu <sub>15</sub> Ru <sub>20</sub> Si <sub>10</sub>	187		[9]
1/1-Al <sub>68</sub> Cu7Ru17Si8	187	~99	[30]

Kimura et al.[5]. It is very difficult to estimate experimentally the structural vacancy densities in stable Al-based quasicrystals and their approximant crystals. In order to estimate the structural vacancy density in quasicrystals AlPdMn and AlPdRe, we have measured the positron annihilation Doppler-broadening spectra by slow positron beam in quasicrystals  $Al_{70.7}Pd_{21.34}Re_{7.94}$ ,  $Al_{71.5}Pd_{20.3}Mn_{8.2}$ , and the approximate crystal  $1/1 \alpha$  -phase  $Al_{68.31}Mn_{21.21}Si_{10.48}$  [16]. The structure of  $1/1 \alpha$  -phase Al-Mn-Si is a bcc stacking of Mackay icosahedra with the "glue" of Al(Si) atoms among them. A Mackay icosahedron is composed of two concentric icosahedra with a vacant center as shown in Figure 2.



Fig.2 A Mackay icosahedron

The presence of a vacant center is consistent with the lifetime ~205  $\pm$  4ps in Table 1. In the case of 1/1  $\alpha$ -phase Al-Mn-Si, it is thought that most thermalized positrons are trapped in the vacant site within the region of a certain radius larger than the size of vacancies due to their high sensitivity. We call it a positron trapping radius. The positron trapping radius in  $1/1 \alpha$ -phase Al-Mn-Si has been determined by starting from the structure [20]. Through fitting the measured S-parameter date for  $1/1 \alpha$  -phase Al-Mn-Si in the calculated one. we estimated that the positron trapping radius is about 4 Å. With the vacant centers of Mackay icosahedra density in icosahedral quasicrystals Al-Pd-Mn and Al-Pd-Re as well as  $1/1 \alpha$  -phase Al-Mn-Si, it is thought naturally that most thermalized positrons are trapped in vacant center sites of Mackay icosahedra in these quasicrystals selectively with the same positron trapping radius as the one in  $1/1 \alpha$  -phase Al-Mn-Si. Namely it is not unreasonable that the trapping radius in icosahedral quasicrystals Al-Pd-Mn and Al-Pd-Re can be approximated as 4Å. The icosahedral quasicrystals AlPdMn and AlPdRe have the structure arranged quasiperiodically with Mackay icosahedral clusters. Analyzing the measured S-parameter, the structural vacancy densities for these quasicrystals can be estimated experimentally [16]. The structural vacancy densities in icosahedral quasicrystals Al71.5Pd20.3Mn8.2 and Al<sub>70.7</sub>Pd<sub>21.34</sub>Re<sub>7.96</sub> have been estimated to be  $5.0 \times$  $10^{20}$  cm<sup>-3</sup>,  $7.7 \times 10^{20}$  cm<sup>-3</sup>, respectively, as shown in Table 2.

Table	2.	The	structural	vacancy	densities	for
quasicrystals AlPdMn and AlPdRe						

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quasicrystals	Structural density (cm <sup>-3</sup> )	vacancy			
Al <sub>71.5</sub> Pd <sub>20.3</sub> Mn <sub>8.2</sub>	$5.0 \times 10^{20}$				
Al <sub>70,7</sub> Pd <sub>21,34</sub> Re <sub>7,96</sub>	$7.7 \times 10^{20}$				

3. COVALENT BONDING NATURE INDUCED WITH STRUCTURAL VACANCIES, sp-d HYBRIDIZATION, AND QUASICRYSTAL-LIKE STATE

In preceding section, it has been shown that covalent bonding nature, which is induced with structural vacancies in icosahedral clusters, plays important role in semiconductor-like transport properties in stable Al-based quasicrystals. In addition, the role of hybridization between the d and sp orbitals near  $E_F$  in the transport property is of significance, since very high resistivity and its strong negative temperature dependences have been observed only in stable Al-based quasicrystals containing transition metal atoms [21]. Now we shall consider the relationship among covalent bonding nature in icosahedral clusters with structural vacancies, sp-d hybridization, and the anomalous transport property in stable Al-based quasicrystals. Recently one of the present authors [22-24] has considerd the transport property in the randomly distributed system of the correlated configurations (the aggregation), in which the nearest distance between each configuration is  $\sim 2\pi/2k_F$  (the quasicrystal-like state). The quasicrystal-like state is regarded as the system composed of the Gaussian correlated distribution of the icosahedral cluster, which includes 2k<sub>F</sub>-phase shift scattering. Taking into account the short mean free path, 15-20Å of the electrons in quasicrystals, the transport property in the quasicrystal-like system might be closely analogous to one in the quasicrystal system.

The 2k<sub>F</sub>-phase shift scattering induces a standing density wave of sp electrons with wave length  $\sim 2 \pi / 2k_F$ . When the high-density region of the standing wave is located on the transition metal atom, the sp electrons hybridize more strongly the d orbitals of the transition metal atoms located in the configuration. Thus the 2k<sub>F</sub>-phase shift scattering and the sp-d hybridization are correlated to each other. In this case, the value of  $\gamma$ , which is defined as  $1/[2 \pi \rho \tau]$ , becomes large. Note that  $\gamma \propto$  $n_{i} M |V_{d,sp}|^{2}$ . Here  $\rho$  is the density of state per spin at the Fermi energy and  $\tau$  is the lifetime of the sp electron wave due to the sp-d hybridization. The matrix element V<sub>sp.d</sub> represents the sp-d hybridization; and n<sub>i</sub> is the density of the aggregation, which is composed of N number of configurations connected in the distance  $\sim 2\pi$  $/2k_{\rm F}$ . The aggregation might be identified with the icosahedral cluster such as the Bergman type and the Mackay type [25,26]. N depends upon the type of material. When the system is amorphous or when many defects are introduced, the aggregation, which is composed of correlated N number of configurations connected with the distance  $\sim 2 \pi / 2k_{\rm F}$ , is broken. As a result, the value of  $\gamma$  will decrease remarkably. Performing an average over the position of the icosahedral cluster, we assume the potential of the icosahedral cluster as a random quantity having a Gaussian  $\delta$ -correlated distribution. Performing average over the position of the icosahedral cluster, we assume the potential of the icosahedral cluster as a random quantity having a Gaussian  $\delta$ -correlated distribution law. Thus, the propagator  $\Gamma(q, \omega_l)$  of the 2k<sub>F</sub>-phase shift scattering with the sp-d hybridization can be introduced as follows:

$$\Gamma(1,\omega_l) \sim \frac{\gamma}{1-\gamma \Pi(q,\omega_l)}$$
(1)

Here  $\omega_l$  is  $2\pi/T$  and *l* is integer.

The thermal Green function  $D(k, \varepsilon_n)$ ,  $(\varepsilon_n = (2n+1)\pi$ T, T being the temperature) is given by

$$D(k,\varepsilon_n) = \left[ i\varepsilon_n - \xi_k + \frac{i}{2\tau} sgn \varepsilon_n \right]^{-1}$$

$$\frac{1}{\tau} \sim 2\pi\rho\gamma$$
(2)

Here  $\zeta_k = k^2 / 2m - E_F$ , and  $\rho$  is the density of state per spin at the Fermi energy.

 $\Pi(q, \omega_l)$  is represented as follows:

$$\Pi(q, \varpi_l) = \sum_{k,k_F} D(k + q - 2\hat{k}_F, \varepsilon_n + \omega_l) D(-k + 2\hat{k}_{F,\varepsilon_n})$$

$$\sim \rho \int_{-\infty}^{\infty} d\xi \int d\Omega$$

$$\times \left[ i\varepsilon_n + i\omega_l - \xi - vq + \frac{i}{2\tau} sgn(\varepsilon_n + \omega_l) \right]^{-1}$$

$$\times \left[ i\varepsilon_n - \xi + \frac{i}{2\tau} sgn(\varepsilon_n) \right]$$
(3)

Here  $\zeta_{-k} = \zeta_{k}$ ,  $v \equiv k/m$  and  $k_F$  is the vector of the Fermi momentum.

The integration over  $d\Omega$  is over the angle of v. If  $\varepsilon_n(\varepsilon_n + \omega_l) < 0$ ,  $\Pi(q, \omega_l)$  is evaluated for small q and  $\omega_l$  as follows:

$$\Pi(q,\omega_l) \sim 2\pi\rho\tau \left[1 - Dq^2\tau - |\omega_l|\tau\right]$$
(4)

Here  $D=v_F^2 \tau/d$  is the diffusion constant.  $v_F$  and d are Fermi velocity and dimension of system, respectively. Thus  $\Gamma(q, \omega_l)$  is represented as

$$\Gamma(q,\omega_l) = \frac{1}{2\pi\rho\tau^2 \left[Dq^2 + |\omega_l|\right]}$$

By using  $\Gamma(q, \omega_l)$ , the replacement  $\omega_l \to i \omega$ , and the Green functions  $D_{\pm}(k)$  we can estimate the conductivity.

(5)

$$\Delta\sigma(\omega) = \frac{e^2}{\pi} \sum_{k} \sum_{k'} (\eta/m)^2 k_x k'_x D_+(k) D_-(k)$$
$$\times D_+(k') D_-(k) \Gamma(k+k'=q,-i\omega)$$
(6)

The  $2k_F$  phase shift scattering with the sp-d hybridization by correlated clusters influences the thermal Green function approximately in the lowest order as follows:

$$D(k,\varepsilon_n) = \left[i\varepsilon_n - \xi_k + \frac{i}{2\tau}sgn\varepsilon_n - \sum_1 - \sum_2\right]^{-1} \sum_{1 \sim -T} \sum_{\substack{\omega l \ q}} D(-k+q,\varepsilon_n + \omega_l)\Gamma(q,\omega_l) \sum_2 \sim -T \sum_{\substack{\omega l \ q}} \sum_q D(k+q,\varepsilon_n + \omega_l)D(q,\omega_l)$$
(7)

 $D(q, \omega_l)$  is the particle-hole correlation function (the diffusion mode), and is represented by  $D(q, \omega_l) \sim 1/2 \pi \rho \tau^2 [Dq^2 + |\omega_{\parallel}]$ .

$$\sum_{q} \sum_{l} \sum_{q} \sum_{q} \frac{2\pi\rho\gamma^{2}}{Dq^{2} + |\omega_{l}|} \times \left[ D(-k+q,\varepsilon_{n}+\omega_{l}) + D(k+q,\varepsilon_{n}+\omega_{l}) \right]$$

Since q and  $|\omega_l|$  are small,  $D(-k+q, \varepsilon_n + \omega_l) + D(k+q, \varepsilon_n + \omega_l)$  for  $|k| \sim k_F$  and  $\varepsilon_n \sim 0$  is represented as  $-4i \tau sgn(\varepsilon_n + \omega_l)$ . Thus

(8)

$$\sum_{n} \sum_{1} \sum_{2} -4i \operatorname{sgn}(\varepsilon_{n}) T \times \sum_{\omega l} \sum_{q} \frac{\gamma}{Dq^{2} + |\omega_{l}|}$$

$$\operatorname{Im}\left[\sum_{1} \sum_{1} \sum_{2} -4 \operatorname{sgn}(\varepsilon_{n}) T \sum_{\omega l} \sum_{q} \frac{\gamma}{Dq^{2} + |\omega_{l}|} \sim \frac{1}{2\tau_{1+2}} \right]$$
(10)

Doing analytic continuation,  $i \varepsilon \to \omega$ , of  $\text{Im}D(k, \varepsilon_n)$ , the spectral function is represented by  $A(k, \omega) \propto \text{Im}\widetilde{D}(k, \omega)$ 

$$=\frac{1/2\tau+\mathrm{Im}(\sum_{1}+\sum_{2})}{\left[\omega-\xi_{k}-\mathrm{Re}(\sum_{1}+\sum_{2})\right]^{2}+\left[1/2\tau+\mathrm{Im}(\sum_{1}+\sum_{2})\right]^{2}}$$
(11)

More exactly we must calculate the conductivity self-consistently, taking into account higher order contribution. When  $[1/2 \tau + Im(\Sigma_1 + \Sigma_2)]$  is much larger than  $[\omega - \zeta_{k} - Re(\Sigma_1 + \Sigma_2)]$  -the band width  $W \propto \exp[\{-[3/(4\pi N_c)]^{1/3}\}$ , the pseudogap will grow near the Fermi energy in the spectrum function, where  $N_c$  is the concentration of carriers. As shown in Figure 1, the low concentration of carriers near the Fermi energy is created through the low hopping rate between neighboring cluster, in which the covalent bonding nature is induced by the structural vacancies. Forthermore, the narrow band width of low concentration of carriers induces the pseudogap near the Fermi energy by using eq (11), as shown in Figure 3.

Defining  $L_{1+2} \sim (D \tau_{1+2})^{1/2}$ , we can estimate the correction  $\Delta \sigma_{1+2}$  to the conductivity in the three-dimensional system by  $\Sigma_1 + \Sigma_2$  as follows:

$$\Delta \sigma_{1+2} = \frac{e^2}{\eta} \frac{1}{L_{1+2}} = \frac{e^2}{\eta} \left[ D \cdot \left( 8 \operatorname{sgn}(\varepsilon_n) T \sum_{\omega} \sum_q \frac{\gamma}{Dq^2 + |\omega_l|} \right)^{-1} \right]^{-1/2}$$
(11)

In the case of  $\varepsilon_n > 0$  (the case of  $\varepsilon_n < 0$  can be treated

similarly) each summation over  $\omega_1$  in equation (11), in the condition of  $\varepsilon_n$  ( $\varepsilon_n + \omega_l$ )>0, is evaluated as follows:

$$\Delta \sigma_{1+2} = \frac{e^2}{\eta} \left[ D \left( \frac{8\gamma}{2\pi^2 D^{3/2}} T \sqrt{2\pi T} \sum_{-\tau^{-1}/(2\pi T) < -\varepsilon_{\pi}/(2\pi T)} \right)^{-1} \right]^{-1/2}$$
(12)

In the region of low temperature, that is  $\tau T \rightarrow 0$ , the temperature dependence of  $\Delta \sigma_{1+2}$  is given as  $\Delta \sigma_{1+2}$  $\propto T^{3/4}$ . More exactly we must calculate the conductivity self-consistently, taking into account the higher order contribution. We have analyzed the conductivities of the metallic Al-Pd-Re quasicrystals by Tamura et al. [27] from the viewpoint of the present theoretical formula. The temperature dependence curve by the  $T^{1/2}$  power law below 10K and the  $T^{3/4}$  power law from 10 to 30K is shown. It appears that the  $T^{3/4}$  power law is similar to predicated  $\overline{T^{3/4}}$  dependence for  $\Delta \sigma_{1+2}$  in eq (12) [28].



Fig.3 A schematic picture of the electron state near the Fermi energy in stable Al-based quasicrystals

# 4. CONCLUSION

It is shown that structural vacancies and sp-d hybridization induce growth

Of pseudogap near the Fermi energy and the anomalous transport properties in stable Al-based quasicrystals.

We have discussed the anomalous transport ptoperties in Al-based quasicrystals-like system, taking into account the sp-d hybridization.

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